MATH 3912 - Assignment 1

1. Compute the determinant of
\[
\begin{pmatrix}
6 & 0 & -8 \\
8 & 3 & 0 \\
6 & 3 & -4
\end{pmatrix}
\]

2. Compute the determinant of the lower triangular matrix
\[
\begin{pmatrix}
a_{11} & 0 & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 \\
a_{31} & a_{32} & a_{33} & 0 \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{pmatrix}
\]

3. Suppose \(A\) is a lower triangular \(n\) by \(n\) matrix as defined below. Show that \(\text{det}(A) = \prod_{i=1}^{n} a_{ii}\), or in other words the determinant of \(A\) is the product of the entries in the main diagonal.
\[
A = \begin{pmatrix}
a_{11} & 0 & \ldots & 0 \\
a_{21} & a_{22} & 0 & \ldots \\
a_{31} & a_{32} & a_{33} & 0 & \ldots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
a_{n1} & a_{n2} & a_{n3} & \ldots & a_{nn}
\end{pmatrix}
\]

4. Find the determinant of the matrix \(V\) defined as follows:
\[
V = \begin{pmatrix}
1 & z_0 & z_0^2 \\
1 & z_1 & z_1^2 \\
1 & z_2 & z_2^2
\end{pmatrix}
\]

5. Solve the following system of equations, if there is a solution:
\[
\begin{align*}
8x_1 - 2x_2 + x_3 &= 1 \\
2x_1 - 8x_2 &= 7 \\
-2x_1 - 6x_2 - 7x_3 &= 2
\end{align*}
\]

6. Suppose \(p(x) = a_0 + a_1x + a_2x^2\) is a quadratic polynomial. Find the coefficients of that polynomial so that
\[
\begin{align*}
p(-1) &= 1 \\
p(0) &= 4 \\
p(1) &= 1
\end{align*}
\]

7. Suppose
\[
\vec{v}_1 = \langle 7, -5, 5 \rangle, \quad \vec{v}_2 = \langle 6, 0, 0 \rangle, \quad \text{and} \quad \vec{v}_3 = \langle -3, 5, 0 \rangle
\]
are three vectors in \(R^3\). Are they linearly independent? What about the vectors
\[
\vec{w}_1 = \langle -3, -6, -7 \rangle, \quad \vec{w}_2 = \langle 0, 2, -15 \rangle, \quad \text{and} \quad \vec{w}_3 = \langle 3, 8, -8 \rangle
\]
How about
\[
\vec{z}_1 = \langle 6, 0, 0 \rangle, \quad \vec{z}_2 = \langle -3, 5, 0 \rangle, \quad \vec{z}_3 = \langle -3, -6, -7 \rangle, \quad \text{and} \quad \vec{z}_4 = \langle 0, 2, -15 \rangle
\]