Panel 1

Last time (A long time ago): we computed

\[ P(z > 0.75) \text{ for } N(0,1) \]

\[ P(6 \leq x \leq 12) \text{ for } N(8.7) \]

\[ P(x \in [a, b]) = \int_a^b f(x) \, dx \]

Conversion from \( N(\mu, \sigma) \) to \( t \)-score:

\[ t = \frac{x - \mu}{\sigma} \]

12 - 2, \( x \sim N(10) \)

\[ \eta^2 \text{-value} = 2 \times \frac{12 - 2}{2} = \frac{10}{2} = 5 \]

Panel 2

Reverse Look-up

\[ P(z > z_0) = 0.25 \]

\[ x \sim N(\mu, \sigma) \]

\[ P(x \leq k) = 0.7 \text{ where } x \sim N(7.5, 10) \]

Conversion from \( t \) to \( N(\mu, \sigma) \):

\[ t = \frac{x - \mu}{\sigma} \]

Panel 3

Central Limit Theorem - everything is normal!

Say we have a distribution of unknown shape, with mean \( \mu \) and std. dev. \( \sigma \).

Suppose we keep selecting samples of size \( n \) and compute the sample mean \( \bar{x} \) each time.

Then, the \( \bar{x} \) have normal distribution with mean \( \mu \) and std. dev. \( \frac{\sigma}{\sqrt{n}} \)

Short Form: \( \bar{x} \sim N(\mu, \frac{\sigma}{\sqrt{n}}) \)

Panel 4

Check in any way for US cars. 2.5% have some defect with some in annual survey, say \( N = 100 \), and check their annual it. Say I get sample mean \( \bar{x} = 23.5 \) and \( s = 3.32 \).

Again: 400 other cars, and \( \bar{x} = 22.9 \) (\( 23.3 \)).

Repeat: 400 cars, and \( \bar{x} = 24.1 \) (\( 23.5 \)).

Known \( \bar{x} \) one \( N(\mu, \sigma^2) \)

Ex. Want to know \( P(\bar{x} \leq 25) = 0.9 \)
Panel 5

Panel 6

Panel 7

Panel 8

Confidence Intervals

To find a 90% confidence interval about population mean $\mu$:

1. Find $\frac{S}{\sqrt{N}}$ (standard error)
2. Compute $1.645 \cdot \frac{S}{\sqrt{N}}$
3. Answer: between $\bar{x} - 1.645 \cdot \frac{S}{\sqrt{N}}$ and $\bar{x} + 1.645 \cdot \frac{S}{\sqrt{N}}$

90% contains $\mu$ on average between $110 \pm 0.9139$ mg, or with 99% contiunity from 109.086 to 110.913
Panel 9

Other common intervals are:

\[
\begin{align*}
80\%: & \quad 0.05 \\
90\%: & \quad 0.015 \\
95\%: & \quad 0.005
\end{align*}
\]

Panel 10

Confidence Interval about \( \mu \)

1. Find \( \frac{s}{\sqrt{N}} \)

2. Multiplex 90\%, 1.645 \( \frac{s}{\sqrt{N}} \)
    95\%, 1.96 \( \frac{s}{\sqrt{N}} \)
    99\%, 2.57 \( \frac{s}{\sqrt{N}} \)

3. Answer:

\[
\bar{x} \pm (\quad)
\]

I am 90\%, 95\%, or 99\% certain.

Panel 11

Ex: The active ingredient of some medication is measured in ppm. A random sample gives:
10, 11, 9.5, 9, 10.9, 11.5, 12.4, 10, 9.8

Find an estimate for the unknown population mean \( \mu \).

The usual confidence interval is 95%.

\[
\bar{x} = 10.052, \quad s = 1.581
\]

\[
\text{SE} = \frac{s}{\sqrt{N}} = 0.577
\]

The upper 1.96 \( 0.577 = 1.152 \)

\[
\mu \text{ is } 10.052 \pm 1.152, \quad \text{or } \mu \text{ is between 8.90 and 11.20}
\]

Panel 12

\text{Simpler: On Phone}

1. \( \bar{x} = 9.83 \)
2. \( s = 1.57 \)
3. \( \bar{x} + 1.96s = 9.83 + 1.96(1.57) \approx 11.3 \)

\( \bar{x} - 1.96s = 9.83 - 1.96(1.57) \approx 8.3 \)

\( \mu \) is between 8.3 and 11.3.
Panel 13

Ex: To hit soldier with new bullets, we need to know their avg head size.

Get 1000 soldiers, find, say, that $\bar{x} = 52 \text{cm}, s = 9.5$

Want to find $\mu$ with 99% certainty.

$$\text{std error} \quad \frac{9.5}{\sqrt{1000}} = 0.2688$$

Multiply: $2.576 \cdot 0.2688 = 0.6823$

$\mu$ equals 52 $\pm 0.6823$ or

$\mu$ between 51.32 and 52.683 with 99% sure it is $\mu$.

Panel 14

Ex: The active ingredient to some medication is measured in ppm. A random sample gives:

10, 11, 9.5, 7, 10.9, 11.5, 12.7, 10, 9.8

Find an estimate for $\mu$.

We seek samples of size 8.

They will have a

Panel 15

Is 90% or 99% bigger?

Want to find or estimate, $\mu$.

With 100% certainty, $\mu$ is between -0.5 to 0.5

99% sure, $\mu$ is between -1 and 1

95% is smaller than (5,11), say (6,10)

90% is smaller still $\approx (9,9)$

1%

$\chi^2$ (8, 8.91)

$P(x^2 \leq 16) = 1$

$P(x^2 > 16) = 0.5$ confidence intervals