**Metric Spaces**

**Exercises**

1. Draw he unit balls for $R\_{2}^{2}$, for $R\_{1}^{2}$, and for $R\_{0}^{2}$. For extra credit, can you *describe* the unit balls in the three n-dimensional spaces, perhaps in words?
2. Show that $R\_{2}^{n}$, $R\_{1}^{n}$, and $R\_{0}^{n}$ really are metric spaces
3. Draw or describe the unit ball for $C^{0}[a,b]$
4. Find the distance between $sin$ and $cos$: $ρ\left(sin,cos\right)$ in $C^{0}\left[0,2π\right]$ and in $C^{2}[0,2π]$
5. Prove the Cauchy-Schwartz Integral inequality *(Hint: try to use a ‘smart’ proof similar to the smart prove of the regular Cauchy-Schwartz inequality)*
6. Show that $C^{2}[a,b]$ is a metric space (hint: use the Cauchy-Schwartz integral inequality)
7. Verify Minkowski’s as well as Hoelder’s inequality for $a = <1,-2,0,1>$ and $b=<-1,0,3,2>$
8. Show that if $a, b\in R^{n}$ are proportional, then Minkowski’s inequality turns into an equality
9. Show that $R^{n}$ with $ρ\left(x,y\right)=\left(\sum\_{j=1}^{n}\left|x\_{j}-y\_{j}\right|^{p}\right)^{\frac{1}{p}}$ , $p>1$,is a metric space, denoted by $R\_{p}^{n}$ (Note that we already have introduced $R\_{p}^{n}$ for $p=0,1$, and $2$.