**Fourier Series – Sample Questions and Quiz**

1. **Theorems and Results**

**Theorem (Fourier Coefficients)**

If then

**Theorem: (Fourier Series)**

Suppose is a piecewise function, i.e. consists of finitely many parts, each of which is continuously differentiable, and all discontinuities are jump discontinuities, then the Fourier series of converges to at all points where is continuous, and to at each jump.

**Theorem (Change of Scale):**

If for with

Then converges uniformly to the function for any piecewise function.

1. **Sample Questions**

**Part 1:**

1. Assuming that the functions below have Fourier series expansions, find them:
   1. b) c)
2. Assuming that the function has a Fourier series, find it and use it to approximate the original function by plotting the function and several N-th partial Fourier series for 1, 2, 3, 4, 5, 10, 20, and 100. Describe in words how the N-th partial Fourier series approximates the original function as N increases.
3. Use a Taylor series and a Fourier series to approximate the function , . Which one works better? How about for f
4. Prove that if had a Fourier series and was an odd function defined on , then , i.e. for all . What about for even functions?

**Part 2:**

1. Use the Fourier series for on to show that (we already knew that by looking at the Taylor series for but our reasoning is different this time around)
2. Use the Fourier series for on to show that
3. Check the Fourier Theorem for , especially at the jump . Verify the convergence behavior

**Part 3:**

1. Find the Fourier series for the ‘golden arches’
2. Compute the Fourier series for

**Sample Quiz Questions**

Here is a sample quiz so that you will know how the in-class quiz on Fourier series might look like. One of the questions below would be marked as option and give you bonus points if completed

1. State Fourier’s Theorem, including the formulas for the Fourier coefficients.
2. Prove that for piecewise and even, i.e. , all Fourier coefficients .
3. Find the Fourier series for
4. Use the fact that to show that
5. Find the Fourier Series for for