**Theorems and Definitions**

* Power Series (Definition and Theorem)
* Obtaining series by (a) differentiation, (b)substitution, (c) differentiation, (d) integration, (e) long multiplication, (f) long division, (g) other kind of voodoo magic
* Taylor Series and Theorem
* Lagrange Remainder
* If $p(x)$ is a polynomial then any Taylor series $T\_{p}(x,c)$ converges to $p$ for any center $c$.
* Not every Taylor series converges to its generating function
* The Most Beautiful Formula in Mathematics

**Part 1**

1. Show that the derivative of the exponential function is again the exponential function
2. Assuming that there is one, find the series for $sin⁡(x)$ and $cos⁡(x)$
3. Verify that $\frac{1}{1-x}=\sum\_{n=0}^{\infty }x^{n}$ by computing the n-th derivative of $\frac{1}{1-x}$
4. Find the derivatives of $sin⁡(x)$ and $cos⁡(x)$
5. Prove that $\sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n}}{n}=ln⁡(2)$, i.e. the alternating harmonic series adds up to $ln⁡(2)$

**Part 2**

1. Find the series expansion for $f\left(x\right)=\frac{2x}{\left(1-x^{2}\right)^{2}}$ and $f\left(x\right)=arctan⁡(x)$
2. Use a series computed above to prove that $π=4(1 – 1/3 + 1/5 – 1/7 + 1/9 -…)$ Note that it is interesting that the complicated transcendental number $π$ is a sum of 1 over plus/minus the odd integers. However, the rate of convergence is rather slow; try to approximate $π$ to, say, 4 digits

**Part 3**

1. Find the first four terms of the power series for $f\left(x\right)=e^{x}∙\sin(\left(x\right))$ by long multiplication. Verify your answer by taking derivatives.
2. Find the first two terms of the power series for $f\left(x\right)=\frac{\cos(\left(x\right))}{e^{x}}$. Verify your answer by writing $f\left(x\right)=\cos(\left(x\right))∙e^{-x}$ and using long multiplication. Finally, verify your answer by taking derivatives.
3. Find the power series for $f\left(x\right)=\frac{1}{e^{x}}$ by long division. Verify your answer by writing $\frac{1}{e^{x}}=e^{-x}$ and finding its power series.

**Part 4**

1. True or false: If $f$ and $g$ are two $C^{\infty }$ functions such that $f^{\left(n\right)}\left(c\right)=g^{\left(n\right)}(c)$ for all $n$ then $f\left(x\right)=g(x)$ for all x.
2. Is every McLaurin series a Taylor series? How about the other way round?
3. Does every Taylor series represent its original function? Prove it or give counter example.
4. Use the above theorem to prove the factor theorem for polynomials, which states that $(x-c)$ is a factor of the polynomial $p(x)$ if and only if $p\left(c\right)=0$

**Part 5**

1. Find the first three nonzero terms in the Taylor series for $tan⁡(x)$ on $[-\frac{π}{4},\frac{π}{4}]$ and estimate the error
2. Find a polynomial approximation for $sin(x)$ on $[-π,π]$ accurate to $\pm 0.005$
3. Prove the theorem we stated in our last segment, i.e. that$ T\_{p}\left(x,c\right)=p(x)$ for any polynomial.
4. Show that $cos⁡(x)$ is equal to its Taylor series for all $x$