Analysis HW

Note Title 3/25/2013

(1) $C[0,2\pi]$ with $\rho(f,g) = \max_{0 \le t \le 2\pi} \{|f(t) - g(t)|\}$ is a metric space. Find

$\rho(\sin(x), \cos(x))$

(2) Prove Schwarz' Inequality

$$\left(\int_a^b f(t)g(t)\,dt\right)^2 \le \left(\int_a^b f(t)^2\,dt\right)\left(\int_a^b g(t)^2\,dt\right)$$

Hint: Start with $\int_a^b\int_a^b (f(s)g(t) - f(t)g(s))^2\,ds\,dt$

(3) Use Schwarz' Inequality to show that

$\rho(f,g) = \left(\int_a^b (f(t) - g(t))^2\,dt\right)^{1/2}$ is a metric defined for all cont functions on $[a,b]$.

(4) We have defined two metric spaces:

$(C[a,b], \rho)$, $\rho(f,g) = \max_{a \le t \le b} \{|f(t) - g(t)|\}$

$(C[a,b], \hat{\rho})$, $\hat{\rho}(f,g) = \left(\int_a^b (f(t) - g(t))^2\,dt\right)^{1/2}$

Find $\rho(\sin(t), \cos(t))$, $\hat{\rho}(\sin(t), \cos(t))$, $\rho(t^2-1, t(t^2-1))$, $\hat{\rho}(t^2-1, t(t^2-1))$ for the interval $[0, 2\pi]$