Analysis 2: HW#4

Note Title 2/18/2013

① Show that if A, B are msble then $\mu(A \cup B) + \mu(A \cap B) = \mu(A) + \mu(B)$

② Find $\mu(C)$, C = Cantor's Middle Third set. (See Def 5.2.12 and Example 5.2.13)

③ Does every set $A \subset \mathbb{R}$ have an outer measure? Is every set $A \subset \mathbb{R}$ measurable? Search the web to find an answer. You don't need to prove anything, just find a reference.

④ Show that

a) $\chi_{A \cap B}(x) = \chi_A(x) \cdot \chi_B(x)$

b) $\chi_{A \cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_A(x) \cdot \chi_B(x)$

c) $\chi_{A^c}(x) = 1 - \chi_A(x)$

⑤ Suppose A, B are msble and $\mu(B) = 0$. Show that $\mu(A \cup B) = \mu(A)$

⑥ Show that if E is msble and $\varepsilon > 0$, then there exists an open set U and a closed set F such that $F \subset E \subset U$ and $\mu(U \setminus F) < \varepsilon$