Analysis 2 HW 3

Note Title 2/7/2013

1. Find $\mu^*(\{x\})$, i.e. the outer measure of a single number.

2. Prove that if $\mu^*(A) = 0$ then $\mu^*(A \cup B) = \mu^*(B)$
   Hint: use the subadditive property of μ^*

3. Find $\mu^*(S)$ for S a countable subset of $\mathbb{R}$.
   Hint: use previous result and property that μ^* is countable subadditive

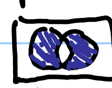
4. What is $\mu^*(\mathbb{Q})$, $\mathbb{Q}$ = rational numbers

5. Show that the interval $[0,1]$ is not countable.

6. For $k > 0$ and $A \subset \mathbb{R}$, define $kA = \{kx : x \in A\}$.
   Show that $\mu^*(A) = k\mu^*(A)$. μ^* is called translation-invariant
   Hint: it is true for intervals!

7. For A, B disjoint, compact intervals, show that $\mu^*(A \cup B) = \mu^*(A) + \mu^*(B)$.
   Note: this is not true in general, but in this case it is.
   Hint: A and B must be separated by a fixed, positive $\delta > 0$.

8. Prove that $\mu^*(A) - \mu^*(B) \leq \mu^*(A \Delta B)$, where Δ is the symmetric difference operator, i.e. $A \Delta B = (A \cup B) \setminus (A \cap B)$



9. Let $A = \mathbb{Q} \cap [0,1]$, i.e. the rationals between 0 and 1. Let $\{A_n\}$ be a finite collection of open intervals covering A. Then,
   $$\sum \ell(A_i) \geq 1$$
   Hint: Try proof by contradiction