Panel 1

Complex HW

① Show that $u(x,y)$ is harmonic and find harm. conjugate:

   a) $u(x,y) = 2x - x^3 + 3xy^2$

   b) $u(x,y) = \dfrac{y}{x^2+y^2}$

② Prove that if $v(x,y)$ and $V(x,y)$ are both harmonic conjugates of $u(x,y)$ then v and V can differ by at most a constant.

③ We defined $e^t = e^x e^{it} = e^x (\cos(y) + i \sin(x))$

   a) solve $e^t = -2$ (all solutions)

   b) show that e^t is entire but $e^{\bar{z}}$ is nowhere diffble

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Panel 2

④ We defined $\log(z) = \ln(r) + i(\theta + 2k\pi)$ and
   $\text{Log}(z) = \ln(r) + i\theta$, where $z = re^{i\theta}$

   a) find $\log(i)$ and $\log(-3)$

   b) show that $\log(i^3) \ne 3\log(i)$ but
   $$\text{Log}(1+i)^2 = 2\,\text{Log}(1+i)$$

⑤ For extra credit, try to determine the values of

   a) 2^i b) i^i Hint: $a^t = e^{t \log(a)}$

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