

Panel 1

Complex HW

① Show that the following functions are entire:

a) $f(z) = 3x + y + i(3y - x)$

b) $f(z) = e^{-y} \sin(x) - i e^{-y} \cos(x)$

② Show that the following functions are nowhere analytic

a) $f(z) = xy + iy$

b) $f(z) = 2xy + i(x^2 - y^2)$

③ Recall from Calc 3 that a function $u(x,y)$ is harmonic if $u_{xx} + u_{yy} = 0$. Prove that if $f(z) = u(x,y) + iv(x,y)$ is analytic and u, v are twice cont. differentiable, then both $u(x,y)$ and $v(x,y)$ are harmonic. Hint: CR

Panel 2

④ Let $f(z) = z^2 + 1/4$. Find

a) $\sigma_f(-\frac{1}{2} + i)$, i.e. the orbit of $-\frac{1}{2} + i$

b) $\sigma_f(-\frac{1}{2} - i)$, i.e. the orbit of $-\frac{1}{2} - i$

c) $\sigma_f(0)$, i.e. the orbit of 0 (first 8 iterates only as decimals)

d) $\sigma_f(2)$, i.e. the orbit of 2 (first 5 iterates only)

e) Which of the above points are in the Fatou set of f ? For some points you need to guess.