Panel 1

Complex Analysis

HW #3

① We know that $\arg(zw) = \arg(z) + \arg(w)$. Is it true for Arg? In other words: is $\mathrm{Arg}(zw) = \mathrm{Arg}(z) + \mathrm{Arg}(w)$ for all $z, w \in \mathbb{C}$?

② Use de Moivre's Formula to derive the following trig identities:

a) $\cos(3\theta) = \cos^3(\theta) - 3\cos\theta\sin^2(\theta)$

b) $\sin(3\theta) = 3\cos^2(\theta)\sin(\theta) - \sin^3(\theta)$

③ Take $z = -1+i$, $w = 2+i$. Show graphically $|z|$, $\arg(z)$, z^2, $1/z$, $z \cdot w$, and $\bar{w}$. Confirm algebraically.

④ Suppose $\omega = e^{i\frac{2\pi}{5}}$. Draw $\omega^0, \omega^1, \omega^2, \omega^3, \omega^4$ and ω^5

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continued =>

Panel 2

⑤ Find both square roots of $(-i)$

⑥ Find the three cube roots of $(-8i)$

⑦ Find the five fifth roots of 1

⑧ Let $\omega_k = e^{i\frac{2k\pi}{n}}$, $k = 0, 1, 2, \ldots n-1$. Show

a) ω_k, $k = 0, 1, \ldots, n-1$ are n n^{th}-roots of unity (1)

b) $1 + \omega_1 + \omega_2 + \ldots + \omega_{n-1} = 0$

Use the Lemma: $1 + z + z^2 + \ldots + z^{n-1} = \frac{1-z^n}{1-z}$ (without proof)

Extra credit: Review the Geometric Series in your Calculus book and prove the Lemma

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