**Complex Analysis Exam 2**

*This is a take-home exam. You may use the book or your notes as you wish, but you* ***must*** *complete each problem on your own. Show all your work (and be neat). Due: last day of finals –* ***no*** *exceptions!*

1. Perform the following integrations along the indicated contours. You can use any method you like.

1. , *C* the unit circle  b) , C the square with corners 1, i, -1, and –i.
2. , C the circle  d) , *C* the circle 
3.  C the circle 

2. Find the Taylor series for each given function centered at the point . Specify the radius of convergence for each series.

a)  b)  c) 

3. Find a Laurent series for the given function centered at the given point  that converges in the specified domain.

a) , , convergent in domain including 

b) , , convergent in domain including 

4. Consider the function  If you were to find the Laurent series centered at converging in the largest annulus  including the point , then what are  and ?

5. Each of the following functions has one or more isolated singularity. Identify each singularity and classify it as removable, pole, or essential. If it is a pole, find its order. Also, find the residue at each singularity.

a)  b)  c) 

6. Use the (complex) Residue Theorem to evaluate . Make sure to justify each step. *Hint*: the answer is 

**Extra credit:** An analytic function  is said to have a *zero of order m* at  if , i.e. the first non-zero coefficient in the Taylor series for *f* is . Suppose  is analytic near  with a zero of order  at  Show that  has a pole of order 1 at . *Hint:* factor what you can from *f(z)*, then work out *f’(z)/f(z)* and use a theorem on what it means to have a pole of order *m* (or 1 in our case).