

Panel 1

Complex HW

① Find the Laurent series centered at  $z_0 = 0$ , and list specifically the value of the coefficient  $a_{-1} = \underline{\hspace{2cm}}$

a)  $f(z) = z^4 \sin(1/z)$       b)  $g(z) = \frac{1}{z^4} \sin(z)$

② The function  $f(z) = \frac{1}{3-z}$  has two series expansions, one for  $|z| < 3$  and another for  $|z| > 3$ . Find them.

③  $f(z) = \frac{-2}{(z-1)(z-3)}$  has 3 series expansions. Find all three series and their domain of convergence.

④ How many series expansions centered at  $z_0 = 0$  does  $f(z) = \frac{1}{e^z - 1}$  have, and where do they converge? →

Panel 2

⑤ Use the theorem on differentiation of power series (which says

$$\text{that } \frac{d}{dz} \sum_{n=0}^{\infty} a_n (z-b_0)^n = \sum_{n=0}^{\infty} a_n \frac{d}{dz} (z-b_0)^n = \sum_{n=1}^{\infty} a_n n (z-b_0)^{n-1})$$

to prove that  $\frac{d}{dz} \sinh(z) = \cosh(z)$

⑥ Use differentiation and/or integration to find the series for

$$f(z) = \frac{1}{(1-z)^2} \quad \text{for } |z| < 1$$