Homework

1. A function \( u(x,y) \) is called harmonic if \( \frac{\partial^2}{\partial x^2} u + \frac{\partial^2}{\partial y^2} u = 0 \)

   Show that \( u(x,y) = e^x \cos(y) \) and \( v(x,y) = e^x \sin(y) \) are harmonic.

2. Are \( u(x,y) = x^2 - y^2 \) and \( v(x,y) = x^3 - 3xy^2 \) harmonic?

   What about \( u(x,y) \cdot v(x,y) \)?

3. For the following functions \( u(x,y) \) and \( v(x,y) \) such that \( f(z) = u + iv \) is analytic:

   a) \( u(x,y) = y^3 - 3x^2y \)  
   b) \( u(x,y) = e^y \sin(x) \)  
   c) \( u(x,y) = \sin(y) \sinh(y) \)

4. Suppose \( f(z) = u(x,y) + iv(x,y) \) is analytic and \( u_x, u_y, v_x, v_y \) are continuous. Show that \( u \) and \( v \) are harmonic.