**Math 3626 Midterm Exam – Review Topics**

You should be familiar with the following concepts:

* Matrix theory (matrix algebra, determinant, Eigenvalues, Eigenvectors, inverse, etc)
* Solving linear systems of equations
* Rotating 2D and 3D vectors; Rotational matrices
* Linear Programming and the Simplex method: Setting up initial tableau, find pivot element, pivot operation, reading off final solution, the “dual” problem
* Complexity of algorithms
* Recursive Algorithms
* 1st order ODE and separable DE
* Higher-order DE’s and converting them to systems of linear Des
* Homogeneous systems of 1st order linear DE’s: Solution in 2D in case of two real Eigenvalues and 2 complex ones; 3 x 3 systems with three different real Eigenvalues; solve initial value problems.

**Sample Problems:**

1. Consider the matrix . Find (a) determinant, (b) Eigenvalues, (c) Eigenvectors, (d) inverse (**no** Mathematica)
2. Same with the 3 x 3 matrix (with or without Mathematica)
3. Solve the following system of equations. If there is no unique solution, state if the system has infinitely many or no solution:
4. Solve the following system of equations (with or without Mathematica)
5. Find the initial tableau for the optimization problem to maximize 2x + 4y + z subject to the constraints x + 3y <= 10, 4x + y + 3z <= 8, x + 6z <= 12 and x,y,z >= 0. Note: you do not need to solve the problem, just set it up properly. Then circle the first pivot element. Finally, perform *one* *complete* pivot operation.
6. Below are two matrices utilizing the simplex method, both using three variables x, y, and z. Which one is done, and what is the answer represented by that matrix? For the matrix that’s not done, circle the next pivot element (but you don’t need to pivot):

1. Solve the following optimization problems:
	1. Maximize 4 x + 2 y subject to 10 x + 5 y <= 50, 12 x + y <= 48, 3 x + 2 y <= 18, x >= 0, and y >= 0.
	2. Maximize 2 x - 3 y + 4 z subject to 4 x - 3 y + z <= 3, x + y + z <= 10, 2 x + y - z <= 10, x >= 0, y >= 0, and z >= 0
2. Solve the problem: minimize x + 3 y but make sure that x + y >= 1 and 4x + 2y >= 3, and of course x >= 0 and y >= 0. Hint: check your notes or the available online resources for the “dual problem”.
3. In class we computed the rotational matrix Rx(x) to rotate around the x-axis by x degrees, Ry(y) to rotate around the y axis by y degrees, and Rz(z) around the z-axis by z degrees. We came up with R(x,y,z) to be:


List the vector <1,2,3> after rotating it by (a) Pi/4 around x, and then by (b) pi/3 about z-axis

1. Consider the following functions. List their order of complexity, i.e. find simple functions g such that f = O(g):
2. What is the order of complexity of (a) adding two matrices, (b) multiplying two matrices, (c) computing the determinant of a matrix using our “standard” algorithm?
3. Solve the following separable 1st order DE: with initial condition . What happens to the solution y as x goes to infinity? Do the same for with
4. Convert the following higher order DE into systems of 1st order DE

Is your ODE system conversion unique?

1. Solve the following system of ODE’s:
2. Solve the following 2nd Order DE: and find all solutions with initial condition y(0) = 1

**Mathematica Functions**

You should be ready to explain the following Mathematica functions and use any of them, as required, for particular programming problems.

**Defining a Function**

:=

**List (Matrix) related**

Length

AppendTo

Join

Drop

[[ ]], [[n]], [[All, n]], [[n,m]]. [[n;m]]

Max

Min

Position

Map

Table

IdentityMatrix

RandomInteger

Eigenvalues

Eigenvectors

**Graphics related**

Graphics

Graphics3D

Polygon

Line

RGBColor

Plot

Plot3D

ListAnimate

RegionPlot

**Programming**

Do

While

If

Print

Timing

**Logical Operators**

==

!=

<, >

<=, >=

&&

||

**Misc**

Inverse

Det

Minimize

Maximize

DSolve