**ODE Homework 1**

1. Convert each linear equation into a system of first order equations.
2. y″ − 4y′ + 5y = 0 2
3. y″′ − 5y″ + 9y = t cos 2t
4. y (4) + 3y″′ − πy″ + 2πy′ − 6y = 11 4

1. Rewrite the system you found in (a) Exercise 1 (a) and (b) Exercise 1 (b), into a matrix-vector equation X’ = A X
2. Convert the third order linear equation below into a system of 3 first order equation using (a) the usual substitutions, and (b) substitutions in the reverse order: x1 = y″, x2 = y′, x3 = y. Deduce the fact that there are multiple ways to rewrite each n-th order linear equation into a linear system of n equations.

y″′ + 6y″ + y′ − 2y = 0

1. Find the Eigenvalues and Eigenvectors for the 2 x 2 matrices manually (you could of course use Mathematica to check your answers but you should be able to do this manually as well)
2. The matrix in 4 (b) is a matrix of a particular type. All matrices of this type have real eigenvalue (for a proof, see Theorem 1 in http://www.quandt.com/papers/basicmatrixtheorems.pdf). Which of the two matrices below is of the same type, and what is that type called? Confirm your guess by finding the Eigenvalues of both matrices.
3. b)
4. Find all solutions to the homogeneous system of linear differential equations X’ = A X, where
5. A is the matrix in 4 (b)
6. A is the matrix in 5 (a)

For a nice, but perhaps a little lengthy, background info about the theory behind this HW, make sure to check: <http://www.math.psu.edu/tseng/class/Math251/Notes-LinearSystems.pdf>