Real Analysis – Homework 2

Note Title 9/1/2012

① Prove that $r: \mathbb{N}_0 \times \mathbb{N}$, $(a,b) \sim (a',b')$ if $ab' = a'b$ is an equivalence relation.

② Show that if we define the domain of the relation in ① to be $\mathbb{N}_0 \times \mathbb{N}_0$, it is no longer an equivalence relation. Hint: r is no longer transitive using $(0,0)$

③ Using the equiv. relation in ①, we define $[(a,b)] \bullet [(a',b')] = [(a\cdot a', b\cdot b')]$. Show this operation is well-defined.

④ Again using ① we define $[(a,b)\cdot(a',b')] = [(ab'+a'b, bb')]$. Show this operation is well-defined as well.

Note The above definitions serve to define $\mathbb{Q}$ (the rationals) - see Thm 1.4.4

5. Actually, the above defines only $\mathbb{Q}^+$ (positive rationals). How could you change the definitions to define all of $\mathbb{Q}$?

6. We defined $\mathrm{card}(A) = \mathrm{card}(B)$ if $\exists$ bijection $f: A \to A$. Prove that $\mathrm{card}(\text{even } \#) = \mathrm{card}(\text{odd } \#)$