**Real Analysis Midterm Sample Questions**

1. The *closure(A)* is the set A together with its boundary points. The *interior(A)* is the set of all interior points of A. True or false: *interior(closure(A)) = A*.
2. If {xn} converges to x and {yn} converges to y, then prove:
   1. xn + yn converges to x + y
   2. xn \* yn converges to x \* y
3. True or false:
   1. If {xn + yn} and {xn} converges, then {yn} converges
   2. If {xn} converges then every subsequence converges
   3. If {xn} diverges then every subsequence diverges
4. Prove that if A and B are compact then is compact. Is that true for countable unions, i.e. is it true that if An is compact for all n, then is compact?
5. Show that any countable set in R has empty interior. How about the converse, i.e. if U is a subset of R with empty interior, then U must be countable?
6. Let and . Prove that {xn} converges and find the limit.
7. Prove that lim (2n + 1) / (n+4) = 2
8. Define Bert’s set by starting with B0 = [0, 1] and removing the middle 5th at each step. Then let . Show that B is uncountable but length of (B) is zero.
9. Can you have the following:
   1. A subset of R that is not bounded but sup(A) is finite
   2. An infinite sequence without accumulation point
10. Give an example of a sequence of numbers such that lim sup(xn), lim inf(xb), sup(xn), and inf(xn) are four distinct numbers.
11. True or false: If converges, then converges. How about without the abs. values, i.e. is it true or false that if converges, then converges.
12. Show that every Cauchy sequence is bounded.
13. True or false: if E is a non-empty subset of R and sup(E) is finite, then there is a sequence in E that converges to sup(E).
14. Show that if E is a non-empty and compact subset of R, then sup(E) is part of E.
15. Find an example of a set A that is not closed and every point is a limit point.
16. Show that an uncountable set must contain at least one of its limit points.
17. Show that if converges to x, then converges to x. Note that the sequence of algebraic means generated from the {xn} is called Cesaro Means.
18. Find a sequence whose Cesaro means converge, but the original sequence does not.
19. Show that if , for all n, converges to x, then the sequence of geometric means also converges to x. *Hint: Look at Cesaro Means and try the natural log.*
20. Test for convergence: