**Math 3515 - Final Exam**

*This is a take-home exam. You may use your notes or the (online) textbook, but you must complete the*

*exam on your own. Please submit the completed exam either in person or via email on the last day of exams.*

1. Give an example for each of the following. You do **not** have to prove your assertion.
   1. A function that is continuous everywhere except at x = 0 and x = 2 (a graph would suffice here)
   2. A function that is not continuous anywhere
   3. A function that is continuous everywhere, but not differentiable at x = 0 and x = 2 (a graph would suffice here)
   4. A function that is differentiable everywhere, but whose derivative is not continuous
   5. A function that is twice differentiable but not three times differentiable
2. Decide whether the following statements are true or false. If false give a counterexample, if true justify your decision.
   1. If the integral  = 0, then f(x) is identically zero on the interval [a, b]
   2. If a sequence of differentiable functions  converges uniformly to *f(x)*, then the limit function *f*  is continuous.
   3. If a sequence of differentiable functions  converges uniformly to *f(x)*, then the limit function *f* is differentiable.
   4. Let . If converges, then converges.
   5. There exists a monotone increasing function that is discontinuous at every irrational number.
   6. If f is differentiable on the interval [a, b], then f is uniformly continuous on that interval.
   7. The union of a finite collection of compact sets is compact.

3. Consider the sequences of functions below. In each case, find the limit function and explain why the convergence cannot be uniform. (Hint: consider the implications of uniform continuity for the limit function, at least in case (a) and (b))

a) , where  for 

b) , where  for 

**Extra Credit:** , where  for 

4. Consider the function . Is this function uniformly continuous on (a) the interval [2, 4], (b) the interval [2, ), or (c) the interval (0, 2) ? Justify your conclusions.

5. Which of the following series is conditionally convergent, absolutely convergent, or divergent ? Justify your conclusions.

a)  b)  c)  ,  (Hint: consider several cases; apply the ratio test)

6. Each of the functions below has a discontinuity at x = 0. Determine whether it is removable, a jump, or essential.

a)  b) 

7. Let *f* be twice continuously differentiable. Prove that if *r*, *s*, and *t* are three roots of the equation *f(x) = 0* with *r < s < t*, then the equation *f’’(x) = 0* has at least one solution in the interval *(r, t)*.

8. Find the center and the radius of convergence of . Also discuss convergence at the endpoints.