Panel 1

Check for convergence

a) $\sum \frac{2^n n^2}{3(n!)}$ Ratio $\frac{2^{n+1}(n+1)^2}{3(n+1)!} \cdot \frac{3n!}{2^n n^2} = \frac{2}{(n+1)} \cdot \left(\frac{n+1}{n}\right)^2 \to 0$

b) $\sum \frac{\sin(n)+3}{n}$ $\sim \sum \frac{1}{n}$ $\Rightarrow$ $\frac{\sin(n)+3}{\cancel{n}} \cdot \frac{\cancel{n}}{1} \to$

c) $\sum (-1)^n \frac{1}{n \ln(n)}$ too tricky

alt. series

$\sum (-1)^n a_n$:

a_n decr., $a_n \to 0$

$\Rightarrow$ series conv.

$\sum \frac{1}{n \cdot \ln(n)} \sim \sum \frac{1}{n n^{0.001}}$

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Panel 2

$\sum \frac{(-1)^n}{n^{3/4}}$ Alt. series ✓

$\frac{1}{n^{3/4}} \to 0$ decr. $\Rightarrow$ conv.

$\sum \left| \frac{(-1)^n}{n^{3/4}} \right| = \sum \frac{1}{n^{3/4}}$ div. $p = 3/4 < 1$. series

$\Rightarrow$ conv. conditionally.

$\sum \frac{(-1)^n}{n^2}$ conv. abs. p-series, $p=2$, compare it

$\left| \frac{(-1)^n}{n^2} \right| = \frac{1}{n^2}$

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Panel 3

Continuity: A function f is cont. at $x=c$ if given any $\varepsilon>0$ $\exists\,\delta$ s.t.

if $|x-c|<\delta \Rightarrow |f(x)-f(c)|<\varepsilon$

OR

f is cont. at $x=c$ if: for every sequence $x_n \to c$ we have $f(x_n) \to f(c)$

Ex: $\lim_{x\to\pi} \sin(x) = \sin(\lim_{x\to\pi} x) = \sin(\pi) = 0$

Analysis: When can you move the limit inside!

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Panel 4

Ex: $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{else} \end{cases}$ is not cont. at any point

$x_n = c + \frac{1}{n}$

Proof: Take any $c \in \mathbb{R}$. $\exists\, x_n \in \mathbb{Q},\ x_n \to c : f(x_n) \to 1$

$\exists\, x_n \notin \mathbb{Q},\ x_n \to c : f(x_n) \to 0$

$x_n = c + \frac{\sqrt{2}}{n}$

Easy: f not cont. at one point, but def. $\forall x$

$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & x < 0 \end{cases}$ Same: f not cont. at countably many pts.

Dirichlet: not cont. at any point.

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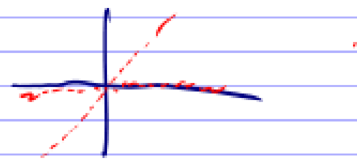
Panel 5

Quest: $f:\mathbb{R}\to\mathbb{R}$, f cont. ONLY at $x=0$.

$$f(x)=\begin{cases} x & \text{if } x\in\mathbb{Q} \\ 0 & \text{else} \end{cases}$$



Cont. at $x=0$: take any sequence $x_n\to 0$. Need $f(x_n)\to f(0)=0$

$|f(x_n)|\le|x_n|\to 0 \quad \Rightarrow$ cont. at 0.

Take any $c\neq 0$. Find $x_n\to c$ s.t. $f(x_n)\not\to f(c)$

HW

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Panel 6

$$f(x)=\begin{cases} 1/q & \text{if } x=p/q\in\mathbb{Q} \\ 0 & \text{else} \end{cases}$$

f is cont. precisely at $x\in\mathbb{Q}$, nowhere else

Is there f s.t. f is cont. precisely at irrationals?

No!

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