

# Calc 3 - Assignment 16

Note Title

10/14/2011

① Find the directional derivative of  $f(x,y)$  at the given point in the direction indicated by  $\theta$ :

a)  $f(x,y) = x^2 y^3 - y^4$ ,  $P(2,1)$ ,  $\theta = \pi/4$

b)  $f(x,y) = x \sin(xy)$ ,  $P(2,0)$ ,  $\theta = \pi/3$

② Find the gradient of  $f$  at the point  $P$ :

a)  $f(x,y) = \frac{y^2}{x}$ ,  $P(1,2)$

b)  $f(x,y,z) = \sqrt{x+yz}$ ,  $P(1,3,1)$

③ Find the directional derivative  $D_{\vec{v}}(f)$ :

a)  $f(x,y) = \ln(x^2 + y^2)$ ,  $\vec{v} = \langle -1, 2 \rangle$

b)  $f(x,y,z) = (x+2y+3z)^{3/2}$ ,  $\vec{v} = 2\vec{j} - \vec{k}$

④ Find the directional derivative of

$f(x,y,z) = xy + yz + zx$  at  $P(1,-1,3)$  in the direction of  $Q(2,4,5)$ .

⑤ Find max. rate of change and its direction:

a)  $f(x,y) = \frac{y^2}{x}$  at  $(2,4)$

b)  $f(x,y,z) = \frac{x+y}{z}$  at  $(1,1,-1)$

⑥ The depth of a lake is given by  
 $z = 200 + 0.02x^2 - 0.001y^3$  measured in feet.

A fisherman in a small boat starts at  $(80, 60)$  and moves toward  $(0, 0)$ . Is the water under the boat getting deeper or shallower?

⑦ The temperature at a point  $(x, y, z)$  is  
 $T(x, y, z) = 200 e^{-x^2 - 3y^2 - 9z^2}$

a) Find the rate of change of  $T$  at  $P(2, -1, 2)$   
in the direction toward  $(3, -3, 2)$

b) In which direction does  $T$  increase  
the fastest?

c) Find the max. rate of increase at  $P$

⑧ Verify that  $u_{xy} = u_{yx}$  for  $u(x, y) = x \sin(x + 2y)$

⑨ Show that  $u = \frac{1}{a^2} t^2 - x^2$  solves the Wave  
Equation  $u_{tt} = a^2 u_{xx}$

3) Which is a solution to Laplace's PDE  $u_{xx} + u_{yy} = 0$

a)  $u = x^2 + y^2$

b)  $u = x^2 - y^2$

c)  $u = \sin(x) \cosh(y) + \cos(x) \sinh(y)$

d)  $u = e^{-x} \cos(y) - e^{-y} \cos(x)$

...