**Surface Integrals**

We have discussed integrals of vector fields over curves, defined as . Now we want to “up” this by one dimension:

**Definition**: If is a surface given by , where are in a region in the xy-plane, and a function, then:

In other words we have that

Compare this with where the curve is given by and , which we discussed before. Note that this integral, just as its new 2D counterpart, has no general geometrical or physical interpretation, but once we define the integration over a vector field, an interpretation will emerge. Also, if the function then

is the **surface area of the surface** , similar to giving the **length of the curve** .

**Example**: Suppose a surface S is given by , and . Find .

The principle problem for this type of problem is to assemble the various pieces that make up this integral; then we can use Mathematica to obtain the answer. Since defines the surface, we have that and , so

But then , where we have of course used Mathematica to evaluate the last double integral.

Two more examples might be helpful:

**Example**: Find the surface area of a sphere, centered at zero with radius R.

According to the comment after the above definition, we know that the surface area is

The equation of a sphere, radius R, is , or to bring it in the form we have:

Is the upper hemisphere, where (x,y) are inside the disk given by . To find the surface area, we need to compute and so that

But then the surface area is

where we switched to polar coordinates to work out the integration.

Note that so far in this course we **proved** that

* area of circle, radius R:
* length of circle, radius R:
* volume of sphere, radius R:
* surface area of sphere, radius R:

Not too bad for a few days work.

**Summary:** To find the surface integral over the surface given by

1. Find the partial derivatives and
2. Compute (and simplify)
3. Replace by in and find the integral (perhaps using Mathematica) as usual using or or polar coordinates