**Quiz 11 - Review**

1. Match the following vector fields:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

1. For the vector field , find
2. Which of the following vector fields are conservative? Find their potential function(s) if they are.

**Recall: Line Integration**

(with respect to x)

(with respect to y)

**Example:** If the curve C is a straight line from (0,1) to (2,5), find the following line integrals:



The line integral with respect to ds has kind of a geometric interpretation:



However, the most common line integral is the integral of a *vector field* over a curve , which also has a very nice interpretation.

**Definition**: The line integral of vector field over a curve C given by is:

or for a 2D vector field and a curve C given by we have

That integral gives the *work* necessary *to move a particle through the field along the path C*

**Note:**  and

**Example:**

1. Find work necessary to move a particle in a straight line from (0,1) to (3,4) through
2. Find , where (i) is a line from (-5,-3) to (0,2) and for (ii) is given by from (-5,-3) to (0,2)
3. Find the work necessary to move a particle along the line from (2,0,0) to (3,4,5) through

Consider the following vector field together with the paths , , and . Determine the signs (pos, neg, or zero) for the following work integrals. Then confirm your answer by working out the three integrals, given that .



Next we will tie everything together by looking at the work done through *conservative* vector fields

**Fundamental Theorem of Line Integration**

If is a *conservative* vector field with potential function , and is a curve from to , then:

Thus, if a vector field is *conservative*, we have *two* ways to find the work integral : (i) you can use the *definition* of the line integral (as long as the path is explicitly given), or (ii) you could find the potential function and then compute the difference . Sometimes one is easier, sometimes the other.

**Example**: Find the following work integrals, using whichever method seems easier.

1. where , C some curve from P(-1,-1) to Q(1,1)
2. where , C upper half circle P(2,0) to Q(-2,0)
3. where C is the V-shaped curve from P(-1,1) to Q(0,0) to R(1,1)