**Line Integration**

Last time:

* a 3D vector field is conservative if: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
* a 2D vector field is conservative if: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Line Integration**

We now introduce a new type of integration that will come in handy when we integrate vector fields. For now, we define:

**Definition:** If , for is a curve , and a function defined in a neighborhood of the curve , then we define the *(line) integral of the function f over the curve C* as:

**Note**: If then is the: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Example**: Find , where C is the straight line from (-1,-1) to (1,3).

To compute line integrals we need to be familiar with “paths”. The following paths occur frequently:

line segment from P to Q circle around 0, radius R graph of graph of

**Example:** Find , where C is a portion of a standard parabola from (0,0) to (1,1) followed by a line segment from (1,1) to (2,3). Also, find , where C is the line segment from (-1,2) to (2,-1)

**Other line integrals**

(with respect to x)

(with respect to y)

**Example:** If the curve C is a straight line from (0,1) to (2,5), find the following line integrals:

**Definition**: The line integral of vector field over a curve C given by is:

That integral gives the *work* necessary *to move a particle through the field along the path C*

**Note:**

**Example:**

1. Find the work necessary to move a particle in a straight line from (0,1) to (3,4) through the field
2. Find , where (i) is a line from (-5,-3) to (0,2) and for (ii) is given by from (-5,-3) to (0,2)
3. Find the work necessary to move a particle along the line from (2,0,0) to (3,4,5) through

Next we will tie everything together by looking at the work done through *conservative* vector fields

**Fundamental Theorem of Line Integration**

If F is a *conservative* vector field with potential function f, and C is a curve from P to Q, then:

Thus, if a vector field is *conservative*, we have *two* ways to find the work integral : (i) you can use the *definition* of the line integral (as long as the path is explicitly given), or (ii) you could find the potential function and then compute the difference . Sometimes one is easier, sometimes the other:

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Quiz 11**

1. Mat0063h the following vector fields:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

1. For the vector field , find
2. Which of the following vector fields are conservative? Find their potential function(s) if they are.