**Vector Fields and Line Integration**

Last time: $div(F)$, $curl\left(F\right)$, $grad(f)$, conservative vector field



**Theorem: (Conservative Vector Fields)**

If a vector field F is conservative, then $curl\left(F\right)=0$. For a 2D vector field, this reduces to the condition $N\_{x}=M\_{y}$. The converse is true in most cases (but not in general).

**Proof:**

**Example**: Which of the following vector fields are not conservative?

1. $\vec{F}\left(x,y\right)=<x, y>$
2. $\vec{F}\left(x,y\right)=<x^{2}+y^{2},2xy>$
3. $\vec{F}\left(x,y\right)=<e^{x}\cos(\left(y\right)),-e^{x}sin⁡(y)>$
4. $\vec{F}\left(x,y\right)=<x^{2}\cos(\left(y\right)),-y^{2}sin⁡(x)>$
5. $\vec{F}(x,y,z)=<y^{2},2xy+e^{3z},3ye^{3z}>$
6. $F\left(x,y,z\right)=<2xy^{2}-2xz^{3},2x^{2}y+3y^{2}z,y^{3}-3x^{2}z^{2}>$

**Example:** Find the potential function, if there is one, for

1. $\vec{F}=<3+2xy,x^{2}-3y^{2}>$
2. $\vec{F}=<x^{2}cos⁡(y),-y^{2}sin⁡(x)>$
3. $\vec{F}=<y^{2},2xy+e^{3z},3ye^{3z}>$
4. $\vec{F}=<2xy^{2}-2xz^{3},2x^{2}y+3y^{2}z,y^{3}-3x^{2}z^{2}>$

Occasionally you can guess (and check) the potential function: The gravity field of an object at $(x,y,z)$ is:

$$\vec{G}\left(x,y,z\right)=\left〈\frac{-x}{\left(x^{2}+y^{2}+z^{2}\right)^{\frac{3}{2}}},\frac{-y}{\left(x^{2}+y^{2}+z^{2}\right)^{\frac{3}{2}}},\frac{-z}{\left(x^{2}+y^{2}+z^{2}\right)^{\frac{3}{2}}}\right〉$$

Its potential function is: