Calc 3 HW 3+4

Note Title 9/8/2011

(1) If $\vec{r} = \langle x, y, z \rangle$ and $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$, describe the set of all points (x, y, z) such that

$$\| \vec{r} - \vec{r}_0 \| = 1$$

(2) Which expression makes sense; where $\cdot$ is dot-product

a) $(\vec{a} \cdot \vec{b})\vec{c}$ b) $\|\vec{a}\|(\vec{b} \cdot \vec{c})$ c) $\vec{a} \cdot (\vec{b} + \vec{c})$

d) $\vec{a} \cdot \vec{b} + \vec{c}$ e) $\|\vec{a}\| \cdot (\vec{b} + \vec{c})$

(3) Find dot product of two vectors if their lengths are 6 and $\frac{1}{3}$ and the angle between them is $\pi/4$.

(4) Find $\vec{a} \cdot \vec{b}$ for

a) $\vec{a} = \langle -2, \frac{1}{3} \rangle$, $\vec{b} = \langle -5, 12 \rangle$

b) $\vec{a} = \langle 4, 1, \frac{1}{4} \rangle$, $\vec{b} = \langle 6, -3, -8 \rangle$

(5) Find angle between $\vec{a}$ and $\vec{b}$ for:

a) $\vec{a} = \langle \sqrt{3}, 1 \rangle$, $\vec{b} = \langle 0, 5 \rangle$

b) $\vec{a} = \langle 4, 0, 2 \rangle$, $\vec{b} = \langle 2, -1, 0 \rangle$

c) $\vec{a} = i + 2j - 2k$, $\vec{b} = 4i - 3k$

(6) Are the vectors orthogonal, parallel, or neither?

a) $\vec{a} = \langle -5, 3, 7 \rangle$, $\vec{b} = \langle 6, -8, 2 \rangle$

b) $\vec{a} = \langle -3, 9, 6 \rangle$, $\vec{b} = \langle 4, -12, -8 \rangle$

c) $\vec{a} = \langle 7, 0, -2 \rangle$, $\vec{b} = \langle 2, 5, 7 \rangle$

(7) For what values of b are the vectors $\langle -6, b, 2 \rangle$ and $\langle b, b^2, b \rangle$ orthogonal?

(8) Find the direction angles (correct to the nearest degree) of $\langle 3, 4, 5 \rangle$

(9)* If $\vec{r} = \langle x, y, z \rangle$, $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, show that the equation

$$(\vec{r} - \vec{a}) \cdot (\vec{r} - \vec{b}) = 0$$

is a sphere. Find its center and radius.

(10)* Show that if $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, then $\vec{u}$ and $\vec{v}$ must have the same length.

(11) Find $\text{proj}_{\vec{a}}(\vec{b})$ and $\text{comp}_{\vec{a}}(\vec{b})$ for
a) $\vec{a} = \langle 3, -4 \rangle$ and $\vec{b} = \langle 5, 0 \rangle$
b) $\vec{a} = \langle 3, 6, -2 \rangle$ and $\vec{b} = \langle 1, 2, 3 \rangle$

Note: $\text{comp}_{\vec{a}}(\vec{b}) = \|\text{proj}_{\vec{a}}(\vec{b})\|$

(12)* The vector $\text{orth}_{\vec{a}}(\vec{b}) = \vec{b} - \text{proj}_{\vec{a}}(\vec{b})$ is called the **orthogonal projection** of $\vec{b}$. Show that $\text{orth}_{\vec{a}}(\vec{b})$ is orthogonal to $\vec{a}$. and draw a picture showing these vectors

(13) Find the work done by a force $\vec{F} = 8i - 6j + 9k$ that moves an object from the point $(0, 10, 8)$ to the point $(6, 12, 20)$ along a straight line.

(14) Find the angle between a diagonal of a cube and one of its edges.

(15) Find the cross product $\vec{a} \times \vec{b}$ for

a) $\vec{a} = \langle 6, 0, -2 \rangle$, $\vec{b} = \langle 0, 8, 0 \rangle$

b) $\vec{a} = \langle 1, 1, -1 \rangle$, $\vec{b} = \langle 2, 4, 6 \rangle$

c) $\vec{a} = \langle t, t^2, t^3 \rangle$, $\vec{b} = \langle 1, 2t, 3t^2 \rangle$

(16) Which expression does <u>not</u> make sense?

a) $\vec{a} \cdot (\vec{b} \times \vec{c})$ b) $\vec{a} \times (\vec{b} \cdot \vec{c})$

c) $\vec{a} \times (\vec{b} \times \vec{c})$ d) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

e) $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$ f) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$

(17) Prove that $\vec{a} \times \vec{a} = 0$ and that $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$ and to $\vec{b}$