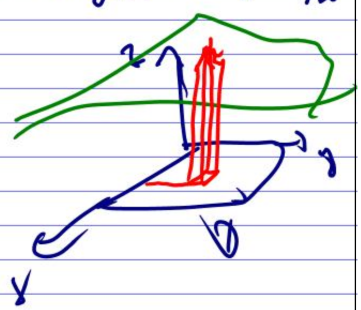
Panel 1

Last Time: Integration $f: D \to \mathbb{R}$, $D \subset \mathbb{R}^2$

Def: $$\iint_D f(x,y)\,dA = \lim_{n\to\infty} \sum_{j,k=1}^{n} f(x_j, y_k)\,\Delta x_j\,\Delta y_k$$



Geometrically: (net) volume under surface $z = f(x,y)$

How to: (Fubini's thm)

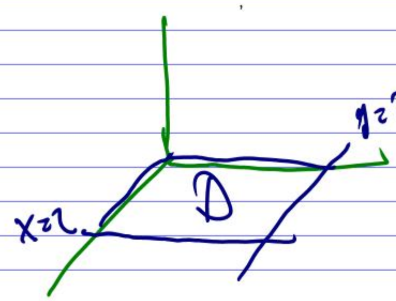
$f: [a,b] \times [c,d] \to \mathbb{R}$

$$\iint_D f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Panel 2

Ex: Find the volume of the solid bounded by $x^2+y^2+z=16$, the planes $x=2$ and $y=3$, and the coordinate planes.

$f(x,y) = z = 16 - x^2 - y^2$



$$V = \int_0^3 \int_0^2 16 - x^2 - y^2 \,dx\,dy$$

$$= \int_0^2 \int_0^3 16 - x^2 - y^2 \,dy\,dx$$

Panel 3

$$\int_0^3\int_0^2 16-x^2-y^2\,dx\,dy = \int_0^3 16x-\frac{1}{3}x^3-xy^2\Big|_{x=0}^{x=2}\,dy$$

$$= \int_0^3 32-\frac{8}{3}-2y^2\,dy =$$

$$= 32y-\frac{8}{3}y-\frac{2}{3}y^3\Big|_0^3 =$$

$$\underline{= 96-8-\frac{54}{3}}$$