Calc 3 - Assignment 30

① Evaluate $\oint_C y\,dx - x\,dy$, C = unit circle

a) directly

b) using Green's theorem

② Use Green's theorem to evaluate:

a) $\oint_C e^y\,dx + 2xe^y\,dy$, C square $x=0, x=1, y=0, y=1$

b) $\oint_C x^2y^2\,dx + 4xy^3\,dy$, C triangle $(0,0), (1,3), (0,3)$

c) $\oint_C (y+e^{\sqrt{x}})\,dx + (2x+\cos(y^2))\,dy$, C region between $y=x^2$ and $x=y^2$

d) $\oint_C \sin(y)\,dx + x\cos(y)\,dy$, C is the ellipse
$$x^2+xy+y^2=1$$

③ We showed in class that $\frac{1}{2}\oint_C x\,dy - y\,dx$ gives area A enclosed by C by letting $M(x,y) = -y$ and $N(x,y) = x$ and applying Green's theorem. Show that also

a) $\oint_C x\,dy = A$

b) $-\oint_C y\,dx = A$

(4) Suppose D is a region in xy-plane bounded by a simple closed path C. Show that the coordinates of the centroid $(\bar{x}, \bar{y})$ of D are

$$\bar{x} = \frac{1}{2A}\oint_C x^2\,dy \qquad \bar{y} = -\frac{1}{2A}\oint_C y^2\,dx$$

(5) Evaluate $\oint_C \vec{F}\,d\vec{r}$ for:

a) $\vec{F} = \langle \sqrt{x} + y^3, x^2 + \sqrt{y} \rangle$, C the curve $y = \sin(x)$ from $(0,0)$ to $(\pi, 0)$ and the line segment from $(\pi, 0)$ to $(0,0)$

b) $\vec{F} = \langle e^x + x^2 y, e^y - xy^2 \rangle$, $C: x^2 + y^2 = 25$

c) $\vec{F} = \langle y - \ln(x^2+y^2), 2\arctan(y/x) \rangle$, C circle $(x-2)^2 + (y-3)^2 = 1$