Calc 3: Assignment 24

① Consider the lamina D bounded by $x = 1 - y^2$ and the coordinate axis in the 1st quadrant with density function $\rho(x,y) = y$. Find the mass of the lamina and the center of mass. Illustrate.

② Sketch the following vector fields

a) $\vec{F}(x,y) = \langle 1, x \rangle$

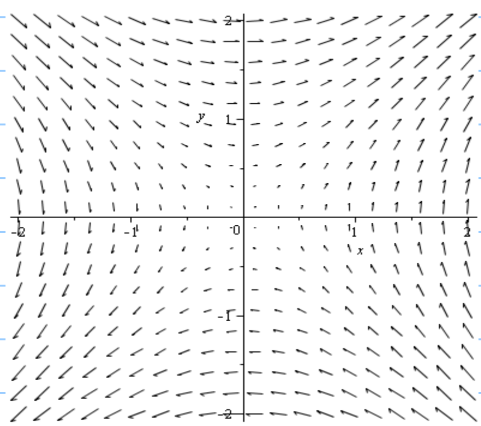
b) $\vec{F}(x,y) = \langle y, \frac{1}{2} \rangle$

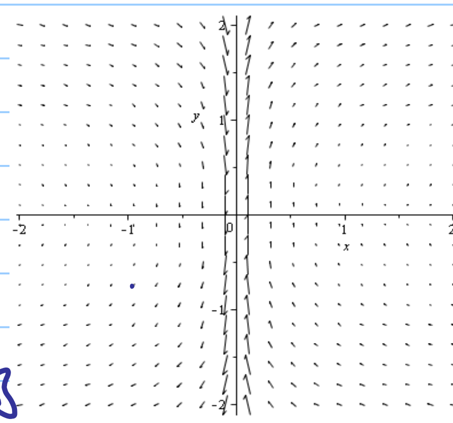
c) $\vec{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}} \langle y, x \rangle$

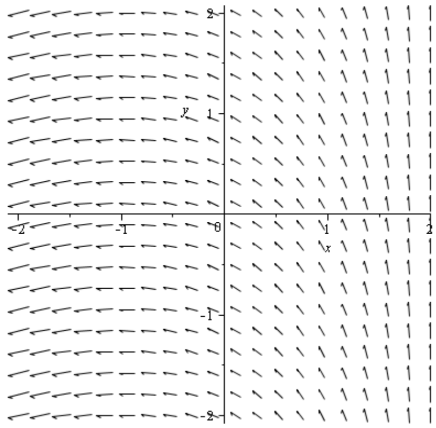
③ Match the vector fields with the plots.

a) $F(x,y) = \langle y, \frac{1}{x} \rangle$ b) $F(x,y) = \langle x-2, x+1 \rangle$

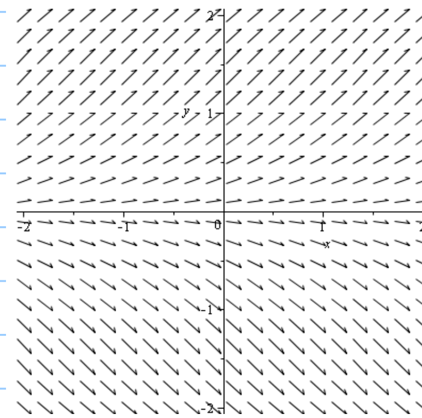
c) $F(x,y) = \langle y, x \rangle$ d) $F(x,y) = \langle 1, \sin(y) \rangle$


A


B



C



D

4) Use Maple to plot $F(x,y) = \langle y^2 - 2xy, 3xy - 6x^2 \rangle$

5) Find the surface area of $f(x,y) = \sqrt{25 - x^2 - y^2}$ over the circle with radius 5. Note that this is the surface area of a ball, radius 5 (the upper half, that is), which we know to be $\frac{1}{2}(4\pi r^2)$.

6) Look at #5 to prove that the surface area of a ball with radius R is $4\pi R^2$. While you are at it, prove that its volume is of course $\frac{4}{3}\pi R^3$

Note: area of circle: πr^2
surface of circle: $2\pi r$

volume of ball: $\frac{4}{3}\pi r^3$
surface of ball: $4\pi r^2$

Make a conjecture as to the relation between an n-dim. ball and its surface!