

Calc 3 - Assignment 16

① Find the directional derivative of $f(x,y)$ at the given point in the direction indicated by θ :

a) $f(x,y) = x^2 y^3 - y^4$, $P(2,1)$, $\theta = \pi/4$

b) $f(x,y) = x \sin(xy)$, $P(2,0)$, $\theta = \pi/3$

② Find the gradient of f at the point P :

a) $f(x,y) = \frac{y^2}{x}$, $P(1,2)$

b) $f(x,y,z) = \sqrt{x+yz}$, $P(1,3,1)$

③ Find the directional derivative $D_{\vec{v}}(f)$:

a) $f(x,y) = \ln(x^2 + y^2)$, $\vec{v} = \langle -1, 2 \rangle$

b) $f(x,y,z) = (x+2y+3z)^{3/2}$, $\vec{v} = 2\vec{j} - \vec{k}$

④ Find the directional derivative of

$f(x,y,z) = xy + yz + zx$ at $P(1,-1,3)$ in the direction of $Q(2,4,5)$.

⑤ Find max. rate of change and its direction:

a) $f(x,y) = \frac{y^2}{x}$ at $(2,4)$

b) $f(x,y,z) = \frac{x+y}{z}$ at $(1,1,-1)$

⑥ The depth of a lake is given by
 $z = 200 + 0.02x^2 - 0.001y^3$ measured in feet.

A fisherman in a small boat starts at $(80, 60)$ and moves toward $(0, 0)$. Is the water under the boat getting deeper or shallower?

⑦ The temperature at a point (x, y, z) is
 $T(x, y, z) = 200 e^{-x^2 - 3y^2 - 9z^2}$

a) Find the rate of change of T at $P(2, -1, 2)$
in the direction toward $(3, -3, 2)$

b) In which direction does T increase
the fastest?

c) Find the max. rate of increase at P

optional

⑧ A function is called homogeneous of degree
 n if $f(tx, ty) = t^n f(x, y)$ for all t

a) Show that $f(x, y) = x^2y + 2xy^2 + 5y^3$ is
homogeneous of degree 3

b) Show that if f is homogeneous of degree n , then

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = n \cdot f(x, y)$$

Hint: Use chain rule to differentiate $f(tx, ty)$ with respect to t .