Calc 3 HW: Distances + Intersections

(1) Find the following distances:

a) between $P(2,-5,5)$ and the plane $x-2y-2z=2$

b) between $P(3,-2,1)$ and $4x-6y-z=5$

c) between $P(1,2,3)$ and $\ell(t)=\langle 2,1,-3\rangle+t\langle 2,2,-1\rangle$

d) between planes $x+2y-z=1$ and $3x+6y-3z=5$

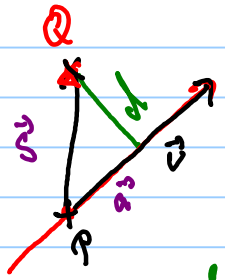
e) between planes $3x+6y-9z=4$ and $x-2y+z=3$

f) between $P(-3,4)$ and line $y=2x-5$

(2) Find line of intersection between planes $x+y+z=1$ and $x-2y+3z=1$

(3)* Show that the distance of Q to a line $\ell(t)=P+t\vec{v}$ is

$$d=\frac{\|\vec{v}\times\vec{PQ}\|}{\|\vec{v}\|}$$



Hint: d is the distance in vect $\vec{a}=\text{proj}_{\vec{v}}(\vec{PQ})$, $\vec{s}=\vec{PQ}$

Use the Fact: $\vec{a}\times(\vec{s}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{s}-(\vec{a}\cdot\vec{s})\vec{c}$

Use this formula to find the distance between

a) $P(1,2,3)$ and $\ell(t)=\langle -1,1,-1\rangle+t\langle 2,3,1\rangle$

b) $P(3,0,4)$ and $\ell(t)=\langle 2,-3,5\rangle+t\langle 1,+3,-1\rangle$

(4) Review the formula for the distance between $P(x_0,y_0,z_0)$ and $ax+by+cz+d=0$ and explain why the distance is zero if P is on the plane.

(5) How do you find the distance between two lines? Hint: the smallest distance is along a vector perp. to both lines. Now project a vector $\vec{PQ}$ with P on line 1 and Q on line 2 onto that vector. Ex: Find the distance between $\ell_1(t)=\langle 2,0,1\rangle+t\langle 1,1,0\rangle$ and $\ell_2(t)=\langle 0,1,1\rangle+t\langle -1,1,-1\rangle$ (Answer is: $3/\sqrt{8}$)