Panel 1

Last Time

Integration in $\mathbb{R}^2$: $\displaystyle\iint_R f(x,y)\,dA = \lim_{n,m\to\infty}\sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i,y_j)\,\Delta x_i\,\Delta y_j$

Fubini's Theorem $\displaystyle\iint_R f(x,y)\,dA$

$$\int_c^d\int_a^b f(x,y)\,dx\,dy \quad \text{or} \quad \int_a^b\int_c^d f(x,y)\,dy\,dx$$

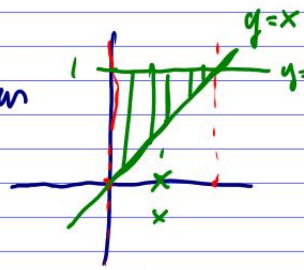
$$R=[a,b]\times[c,d]$$

Panel 2

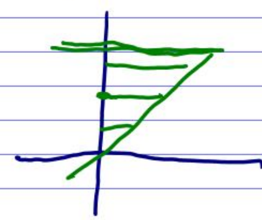
$$\int_0^1\int_x^1 \sin(y^2)\,dy\,dx$$

Stuck: $\sin(y^2)$ has no antideriv.

$y=x$



$$\int_0^1\int_0^y \underline{\sin(y^2)}\,dx\,dy =$$



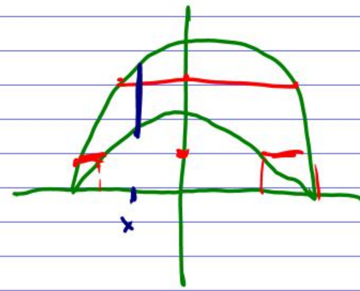
$$\int_0^1 \left[\, x\sin(y^2)\Big|_{x=0}^{x=y} \right] dy =$$

$$\int_0^1 y\sin(y^2) - 0\,dy = \int_0^1 y\sin(y^2)\,dy = -\frac{1}{2}\cos(y^2)\Big|_0^1 =$$

$$= -\frac{1}{2}\cos(1)+\frac{1}{2} = \underline{\frac{1}{2}\left(1-\cos(1)\right)}$$

Panel 3

$$\iint_R f(x,y)\,dA$$



$$\iint dx\,dy$$ 3 integrals!!

$$\iint dy\,dx$$ 1 integral

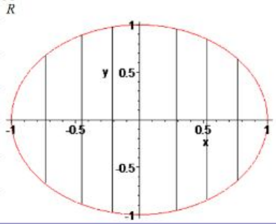
preferred!

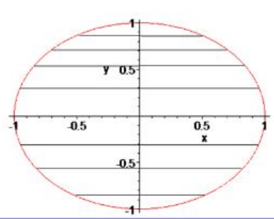
Panel 4

Name:

Quiz

(1) The pictures below show to different ways that a region R in the plane can be covered. Which picture corresponds to the integral $\iint_R f(x,y)dxdy$





(2) Suppose you want to evaluate $\iint_R f(x,y)dA$ where R is the region in the xy plane bounded by $y=0$, $y=2-x^2$, and $y=x$.

According to Fubini's theorem you could use either the iterated integral $\iint f(x,y)dxdy$ or $\iint f(x,y)dydx$ to evaluate the double integral. Which version do you prefer? Explain. You do not need to actually work out the integrals.

Panel 5

(1) Evaluate the following integrals

a) $\int_0^1 \int_0^2 xy^2 dxdy$

b) $\int_0^2 \int_{x^2}^x \left(x^2 + 2y\right) dydx$

Panel 6

Ex: Find $\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$

Done already!!!

Panel 7

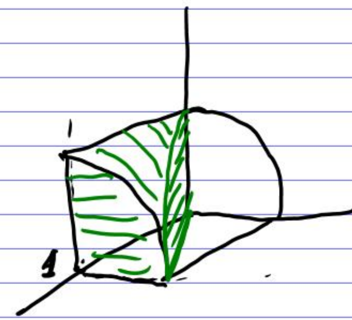
$$\int_0^2 \int_0^1 x^2 y \, dx \, dy$$ Easy

$$\int_0^1 \int_0^y \sqrt{1-y^2} \, dy \, dx$$ no can do

$$\int_0^1 \int_0^y \sqrt{1-y^2} \, dx \, dy$$

$z = \sqrt{1-y^2}$

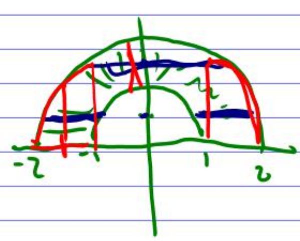
$z^2 + y^2 = 1$



Panel 8

But there are some integral where all tricks (so far) don't work:

$\iint_D (3x + 4y^2) \, dA$ where D is region in upper half plane bounded by $x^2+y^2=1$ and $x^2+y^2=4$



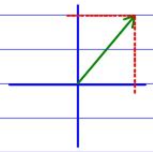
$\int_0^2 \int$ dxdy : 3 integrals

$\iint$ dydx 3 integrals

Polar coordinates

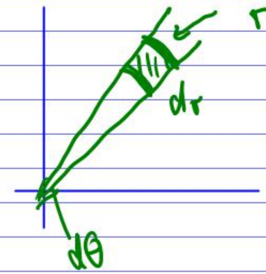
Panel 9

Solution. Polar Coordinates

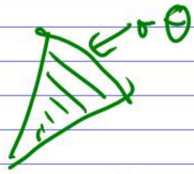


$x = r\cos\theta$

$y = r\sin\theta$



Area is $r\,dr\,d\theta$

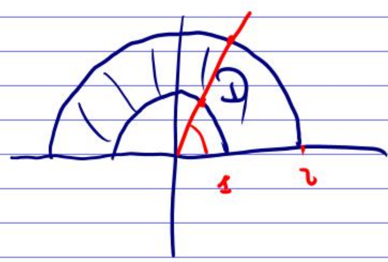


Thus:

$$\iint_R f(x,y)\,dA \begin{cases} \iint f(x,y)\,dx\,dy \\ \iint f(r,\theta)\,r\,dr\,d\theta \end{cases}$$

Panel 10

$\iint_D (3x+4y^2)\,dA$ where D is region in upper half plane bounded by $x^2+y^2=1$ and $x^2+y^2=4$



$$\iint_D 3x+4y^2\,dA =$$

$$\int_0^{\pi}\int_1^2 \left(3r\cos(\theta)+4r^2\sin^2(\theta)\right) r\,dr\,d\theta$$

$x = r\cos\theta$

$y = r\sin\theta$

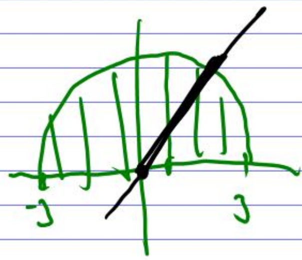
$dA = dx\,dy = r\,dr\,d\theta$

Panel 11

Ex: $$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sqrt{x^2+y^2}\,dy\,dx =$$

$y=\sqrt{9-x^2}$

$x^2+y^2=9$

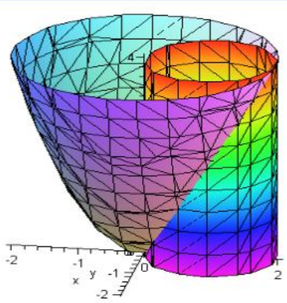


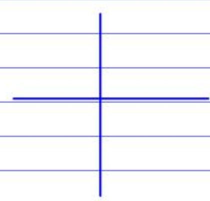
$$= \int_{0}^{\pi}\int_{0}^{3} \sqrt{r^2}\; r\,dr\,d\theta$$

$$= \int_{0}^{\pi}\int_{0}^{3} r^2\,dr\,d\theta = \underline{9\pi}$$

Panel 12

Ex: Volume under $z=x^2+y^2$, inside $x^2+y^2=2x$, above xy-plane





```
with(plots):
implicitplot3d({z=x^2+y^2, x^2+y^2=2x}, x=-2..2, y=-2..2, z=0..4);
```