Panel 1

Last Time

$$\iint_R f(x,y)\,dA = \lim_{n,m\to\infty} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\,\Delta x_i \Delta y_j$$

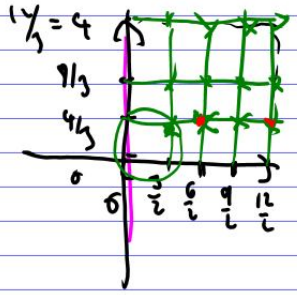
Fubini's Thm (How to integrate in $\mathbb{R}^2$)

If $R = [a,b]\times[c,d] \subset \mathbb{R}^2$, and f is contin. on R, then

$$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$$= \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Panel 2



$f(x,y) = xy$

$$\iint_R f(x,y)\,dA \approx \sum\sum f(x_i, y_j)\,\Delta x_i \Delta y_j$$

$$f\left(\frac{3}{4}, \frac{4}{4}\right)\frac{3}{4}\cdot\frac{4}{4} + f\left(\frac{6}{4}, \frac{4}{4}\right)\frac{3}{4}\cdot\frac{4}{4} + f\left(\frac{9}{4}, \frac{4}{4}\right)2 + f\left(\frac{12}{4}, \frac{4}{4}\right)2 +$$

$$f\left(\frac{3}{4}, \frac{8}{4}\right)2 + f\left(\frac{6}{4}, \frac{8}{4}\right)2 + f\left(\frac{9}{4}, \frac{8}{4}\right)2 + f\left(\frac{12}{4}, \frac{8}{4}\right)2 +$$

$$= 2$$

Panel 3

$$\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dA = \int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dx\,dy = \int_0^1 y\left(\int x\left(x^2+y^2\right)^{1/2}dx\right)dy$$

$$u = x^2+y^2$$

$$\frac{du}{dx} = (2)x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$$= \int_0^1 y\left(\frac{1}{2}\int_{y^2}^{1+y^2} u^{1/2}\,du\right) = \int_0^1 \frac{1}{2}y\cdot\frac{2}{3}u^{3/2}\Big|_{y^2}^{1+y^2}$$

$$= \int_0^1 \frac{1}{3}y\left(1+y^2\right)^{3/2} - \left(y^2\right)^{3/2}dy =$$

$$\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx = \frac{1}{3}\int_0^1 y\left(1+y^2\right)^{3/2} - y^3\,dy =$$

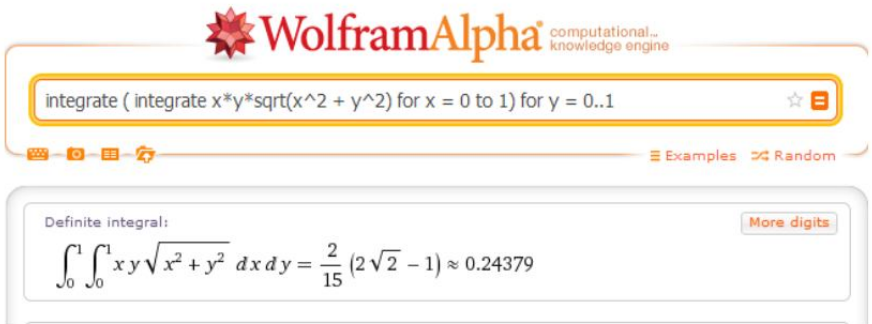
Panel 4

Maple

$$Int\left(Int\left(x\cdot y\cdot \mathrm{sqrt}\left(x^2+y^2\right), x=0..1\right), y=0..1\right) = int\left(int\left(x\cdot y\cdot \mathrm{sqrt}\left(x^2+y^2\right), x=0..1\right), y=0..1\right)$$

$$\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dx\,dy = -\frac{2}{15}+\frac{4}{15}\sqrt{2}$$

WA



Panel 5

Ex) Find the volume of the solid bounded by $x^2+y^2+z=16$, the planes $x=2$ and $y=2$, and the coordinate planes.

$z = 16-x^2-y^2$, $x \in [0,2]$, $y \in [0,2]$

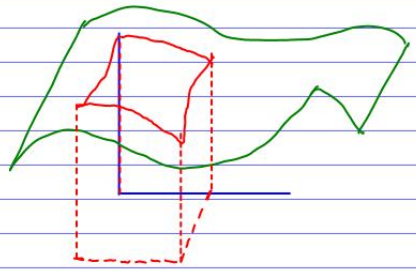
$$V = \int_0^2\int_0^2 16-x^2-y^2\,dx\,dy = \int_0^2 \left(16x - \tfrac{1}{3}x^3 - y^2x\Big|_{x=0}^{x=2}\right)dy$$

$$= \int_0^2 32 - \tfrac{8}{3} - 2y^2\,dy = 32y - \tfrac{8}{3}y - \tfrac{2}{3}y^3\Big|_0^2$$

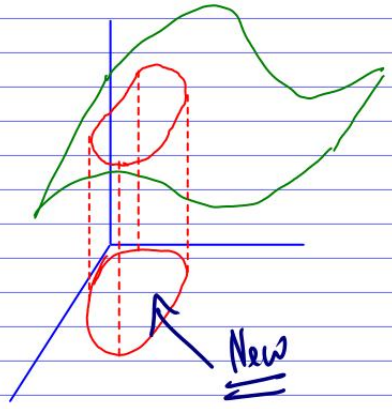
$$= 64 - \tfrac{16}{3} - \tfrac{2}{3}\cdot 8$$

Panel 6

In $\mathbb{R}$ all we ever did was integrate over intervals $[a,b]$. How about in $\mathbb{R}^2$?



Fubini



Panel 7

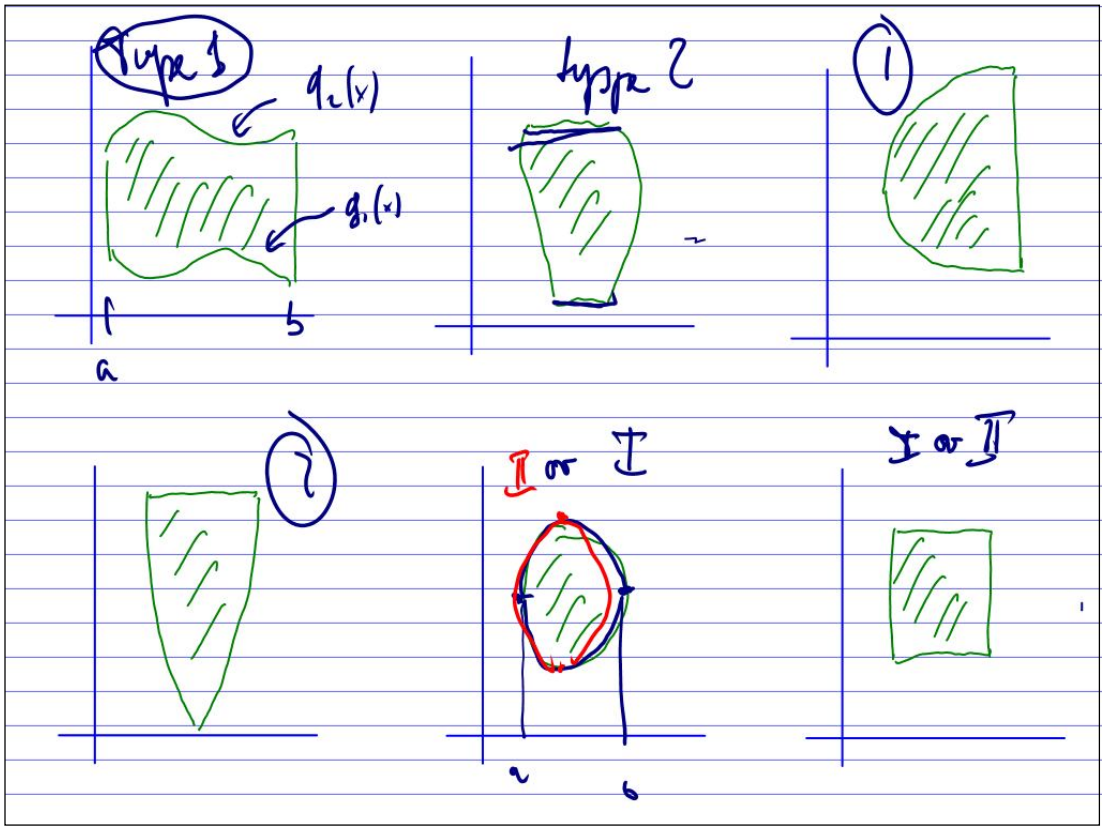
Type 1 Region: $D = \{(x,y): a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$

Type 2 Region: $D = \{(x,y): c \le y \le d,\ f_1(y) \le x \le f_2(y)\}$

Type 1: $$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

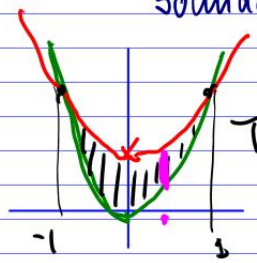
Type 2: $$\iint_D f(x,y)\,dA = \int_c^d \int_{f_1(y)}^{f_2(y)} f(x,y)\,dx\,dy$$

Panel 8



Panel 9

Ex: Find $\iint_D (x+2y)\,dA$ where D is the region bounded by $y = 2x^2$ and $y = 1+x^2$



$2x^2 = 1+x^2$

$x^2 = 1$

$x = \pm 1$

Type I: $\int_{-1}^{1} \int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx = \frac{32}{15}$

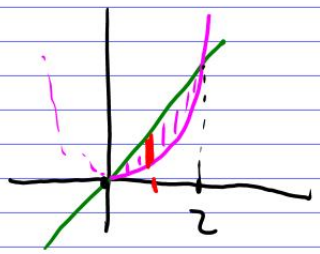
(Maple)

Panel 10

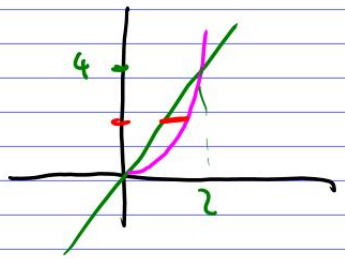
Ex: Volume under $z = x^2+y^2$ above $y = 2x$ and $y = x^2$

$2x = x^2 \Rightarrow x^2 - 2x = 0$

Graph domain in (xy) plane



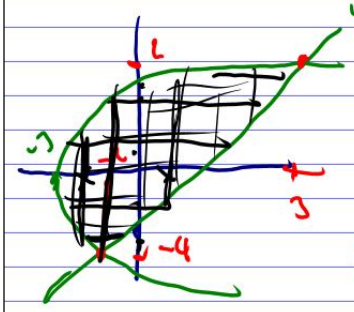
Type I: $\int_0^2 \int_{x^2}^{2x} x^2+y^2\,dy\,dx$



Type 2: $\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} x^2+y^2\,dx\,dy$

Panel 11

Ex: Find $\iint_D xy\,dA$ where D is bounded by $y = x-1$ and $y^2 = 2x+6$. Should you $\iint xy\,dx\,dy$ or $\iint xy\,dy\,dx$?



2 integrals $\iint dy\,dx$

$y = 2x+6$ (1 integral) $\int_{-4}^{2}\int dx\,dy$

HW

$x = y+1$

$x = \frac{1}{2}y^2 - 3$

$y^2 - 6 = 2x$

$\frac{1}{2}y^2 - 3 = x$

$(y+4)(y-2) = 0$

$y+1 = \frac{1}{2}y^2 - 3$

$2y - 2 = y^2 - 6$

$y^2 - 2y - 8 = 0$

Panel 12

Ex: Find $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$

HW

Tricky!