Calc 3: Assignment # 10

Note Title 9/24/2011

① Show that $\frac{d}{dt}\left(\vec{r}(t)\times\vec{r}\,'(t)\right) = \vec{r}(t)\times\vec{r}\,''(t)$

② Find the unit tangent, the normal, and the curvature:

a) $\vec{r}(t) = \langle t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t)\rangle$, $t > 0$

b) $\vec{r}(t) = \langle t, \frac{1}{2}t^2, t^2\rangle$

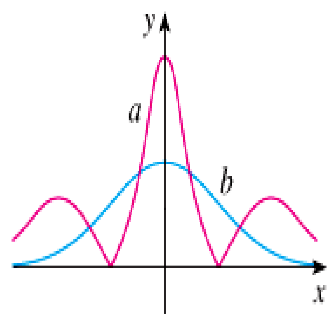
③ Find the curvature:

a) $\vec{r}(t) = (t, t, 1+t^2)$

b) $\vec{r}(t) = \langle e^t\cos(t), e^t\sin(t), t\rangle$

④ Find a formula for the curvature of a function $y = f(x)$. Hint: write the function in parametric form, then embed in $\mathbb{R}^3$. Then compute the curvature using cross products.

⑤ Which of the following curves is the original function $f(x)$, which is its curvature as a function:



(6) Find the vectors T, N, and B for:

a) $\vec{r}(t) = \langle t^2, \frac{2}{3}t^3, t \rangle$ at $(1, \frac{2}{3}, 1)$

b) $\vec{r}(t) = \langle \cos(t), \sin(t), \ln(\cos(t)) \rangle$ at $(1, 0, 0)$

(7) Show that the circular helix

$\vec{r}(t) = \langle a\cos(t), a\sin(t), bt \rangle$, $a > 0, b > 0$, has constant curvature.

(8) Find the unit tangent vector $\vec{T}(t)$:

a) $\vec{r}(t) = \langle 2\sin(t), 5t, 2\cos(t) \rangle$

b) $\vec{r}(t) = \langle t^2, \sin(t) - t\cos(t), \cos(t) + t\sin(t) \rangle$, $t > 0$

c) $\vec{r}(t) = \langle \sqrt{2}t, e^t, e^{-t} \rangle$