Panel 1

Fund. Thm. of Line Integration:

$$\int_C \vec{F}\, d\vec{r} = f(b) - f(a) \qquad \text{if } f \text{ is pot. function of } \vec{F}$$

Thm: If $\vec{F}$ is such that all partial derivs. are cont. in a simply connected domain, then

$\vec{F}$ is conservative $\Leftrightarrow$ $\text{curl}(\vec{F}) = 0$ (3D)

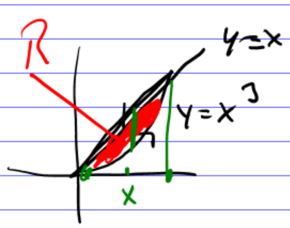
$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$ (2D)

Green: $\oint_C \vec{F}\, d\vec{r} = \oint_C M\,dx + N\,dy = \iint_R N_x - M_y \, dA$ (2D)

1

Panel 2

$$\int_C y^3\, dx + (x^3 + 3xy^2)\, dy = \iint_R 3x^2 + 3y^2 - 3y^2 \, dA =$$



$$= \iint_R 3x^2 \, dA =$$

$$\int_0^1 \int_{x^3}^{x} 3x^2 \, dy\, dx$$

2

Panel 3

Last Time $\vec{F} = \langle M, N, 0 \rangle$

Alternate Version of Green: $\oint_C M\,dx + N\,dy = \iint_R N_x - M_y \, dA = \iint_R \text{curl}(F) \cdot \vec{k} \, dA$

$\left(\int_C f(x,y)\, ds = \int_a^b f(x, g(x)) \sqrt{1 + (g')^2} \, dx \right)$

Surface integral: $\iint_S g(x,y,z) \, dS = \iint_R g(x, y, f(x,y)) \sqrt{1 + f_x^2 + f_y^2} \, dA$

$S: z = f(x,y),\ (x,y) \in R$

Flux integral: $\iint_S \vec{F} \cdot \vec{n} \, dS$ is the flux of $\vec{F}$ through S per unit of time

3

Panel 4

S a surface defined by $z = f(x,y)$, $(x,y) \in D \subset \mathbb{R}^2$. Then S has normal vector $\vec{n} = \langle -f_x, -f_y, 1 \rangle \cdot \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}$

$\vec{F}$ a 3D vector field. Then Flux Integral is

$\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_S \langle M, N, P \rangle \cdot \vec{n} \, dS =$

$= \iint_S (-M f_x - N f_y + P) \cancel{\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}} \cdot \cancel{\sqrt{1 + f_x^2 + f_y^2}} \, dA$

$= \iint_D -M f_x - N f_y + P \, dA \qquad (z = f(x,y))$

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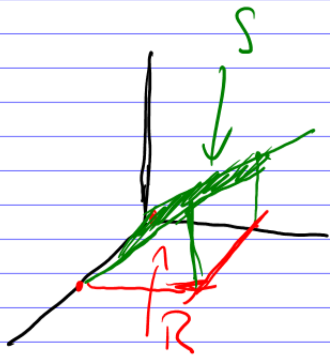
Panel 5

Ex: Let S be the surface defined by $z = y = f(x,y)$ where $(x,y) \in [0,1] \times [0,1]$. Find

$$\iint_S x^2 + y^2 + z^2 \, dS =$$

$$\iint_R x^2 + y^2 + (y^2) \sqrt{1 + \underbrace{0}_{f_x} + \underbrace{1}_{f_y}} \, dA =$$



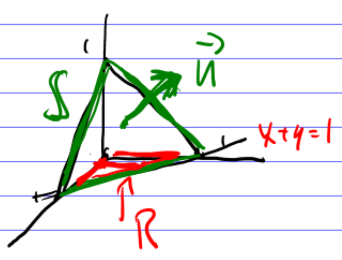
$$\int_0^1 \int_0^1 x^2 + 2y^2 \sqrt{2} \, dy \, dx =$$

Maple

5

Panel 6

Ex: Let S be the surface defined by $x + y + z = 1$ sold by the coordinate planes and let $\vec{F}$ be the vector field $\vec{F} = \langle \overset{M}{x}, \overset{N}{xy}, \overset{P}{xyz} \rangle$. Find the flux of $\vec{F}$ through S.



$$\iint_S \vec{F} \cdot \vec{n} \, dS = \iint_R -Mf_x - Nf_y + P \, dA$$

$$= \int_0^1 \int_0^{1-x} x + xy + xy(1-x-y) \, dy \, dx =$$

Maple

$13/60$

$x + y + z = 1$

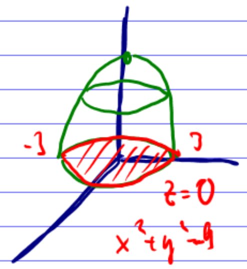
$z = 1 - x - y$, $f_x = -1$, $f_y = -1$, $\vec{n} = \langle \underbrace{1}_{-f_x}, \underbrace{1}_{-f_y}, 1 \rangle \frac{1}{\sqrt{3}}$

6

Panel 7

Ex: Let S be $z = 9 - x^2 - y^2$, $z \geq 0$ and $\vec{F} = \langle 3x, 3y, z \rangle$.

Find flux of $\vec{F}$ through S.



$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint_R -Mf_x - Nf_y + P\, dA$$

$$= \iint_R (3x)(2x) + (3y)2y + (9 - x^2 - y^2)\, dA$$

$$= \iint_R 6x^2 + 6y^2 + 9 - x^2 - y^2\, dA =$$

$f_x = -2x$

$f_y = -2y$

$$= \iint_R 9 + 5x^2 + 5y^2\, dA$$

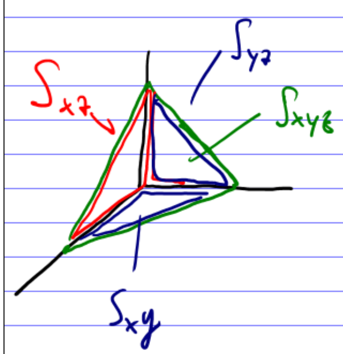
$$= \int_0^{2\pi}\int_0^{3} (9 + 5r^2)\, r\, dr\, d\theta$$

Maple

7

Panel 8

Ex: Let S be the closed surface bounded by $x + y + z = 1$ and by the coordinate planes. Let $\vec{F}$ be the vector field $\vec{F} = \langle x, xy, xyz \rangle$, as before. Find flux of $\vec{F}$ through the closed surface S:



$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint_{S_{xyz}} + \iint_{S_{xy}} + \iint_{S_{yz}} + \iint_{S_{zx}}$$

$$\oint \vec{F}\, d\vec{r}$$

8

Panel 9

The Divergence Theorem (Gauss' Theorem)

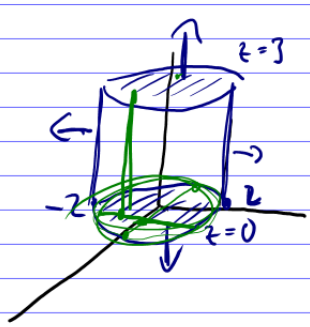
Let Q be a region in $\mathbb{R}^3$ bounded by a closed surface S with outward normal $\vec{n}$. If $\vec{F}$ is a 3D vector field with cont. derivatives, then

$$\oiint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q div(\vec{F})\, dV$$

$$\oint F\, dr = \iint curl(\vec{F})\cdot\vec{n}\, dA$$

Panel 10

Ex: Let S be the surface defined by $x^2+y^2=4$, $z=0$, and $z=3$. If $\vec{F}=\langle x^3, y^3, z\rangle$ find flux of $\vec{F}$ through S.



$$\oiint_S \vec{F}\cdot\vec{n}\, dS = \iiint_Q div(\vec{F})\, dV =$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{+\sqrt{4-x^2}}\int_0^3 3x^2+3y^2+1\, dV = \quad dz\,dy\,dx$$

$$div(\vec{F}) = \vec{F}\cdot\nabla = F_x + F_y + F_z$$

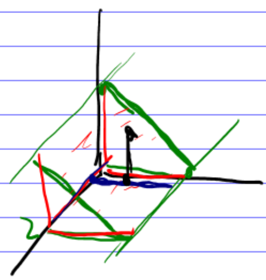
$$= \int_0^{2\pi}\int_0^2\int_0^3 (3r^2+1)\, r\, dz\, dr\, d\theta = \quad dx\,dy$$

Maple

Panel 11

Ex: Q is the region bounded by xz-plane, xy-plane, the planes $x=0$ and $x=2$ as well as $\underline{y+z=1}$

Find
$$\iiint_Q xy\,dV = \int_0^2\int_0^2\int_0^{1-y} xy\,dz\,dy\,dx$$



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Panel 12

Ex: Q is the region bounded by the three coordinate planes and by $x+y+z=1$. Find
$$\iiint_Q z\,dV$$

easy HW

12

Panel 13

Ex: Q region bdd by $z = 4 - x^2$, $y + z = 5$, xy and xz - planes. Find

$$\iiint_Q 4x^2 \, dV$$

tricky HW

13

Panel 14

Ex: Q region bdd by $z = 4 - x^2$, $y + z = 5$, xy and xz - planes.

Let $\vec{F} = \langle x^3 + \sin(z), x^2 y + \cos(z), e^{x^2 + y^2} \rangle$, find $\iint_R \vec{F} \cdot \vec{n} \, dS$

see before HW

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