Panel 1

Last time

Vectors

add + subtract (graphically)

norm

unit vectors (basic units)

dot prod.

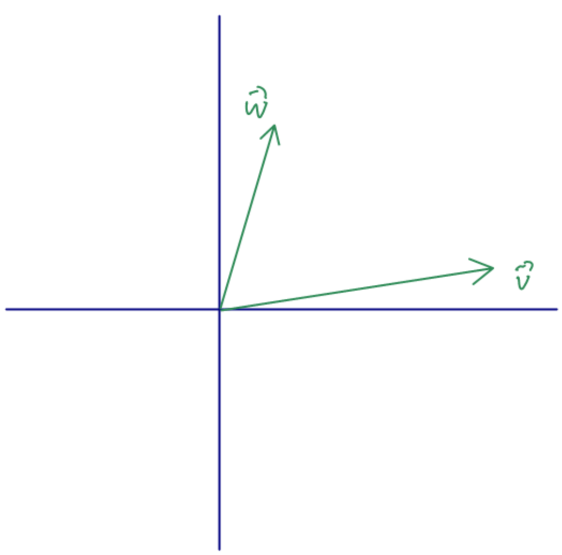
angles

Homework:

Page 841: #6, 13, 17, 18, 19, 21, 23, 24, 26, 28, 29, 31, 39
Page 848: #1, 3, 5, 7, 14, 15, 18, 24, 25, 26, 57

1

Panel 2



Draw in

Red: $\vec{v}+\vec{w}$

Blue: $\vec{w}-\vec{v}$

2

Panel 3

$\vec{v} = \langle 1, 2, 3 \rangle$, $\vec{w} = \langle 5, -1, 2 \rangle$

Find: $\vec{v} + \vec{w}$

$\vec{v} \cdot \vec{w}$

3

Panel 4

If $\vec{v}$ and $\vec{w}$ are perpendicular, then

a) $\vec{v} \cdot \vec{w} = 1$

b) $\vec{v} \cdot \vec{w} = 0$

c) $\vec{v} \cdot \vec{w} = ?$

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Panel 5

Hot dogs: $\langle a,b,c\rangle \cdot \langle 2, 1.5, 1\rangle = 2a + 1.5b + 1c$

#14

a = # hamb., \$2 each

b = # dogs, \$1.50 each

c = # soda, \$1 each

#24 $u = \langle -3,9,6\rangle$, $v = \langle 4,-12,-8\rangle$

ortho; perp; right angle: $u \cdot v = 0$? No

Yes parallel: $k \cdot \langle -3,9,6\rangle \stackrel{?}{=} \langle 4,-12,-8\rangle$

$-3k = 4 \Rightarrow k = -4/3$

$-4/3 \cdot 9 = -12$ ✓

$-\frac{4}{3} \cdot 6 = -8$ ✓

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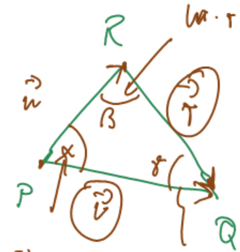
Panel 6

#25 $P(1,-3,-2)$

$Q(2,0,-4)$

$R(6,-2,-5)$

$\vec{w} \cdot \vec{r} \stackrel{?}{=} 0$



$\vec{w} \cdot \vec{v} \stackrel{?}{=} 0$ $\quad \vec{v} \cdot \vec{r} \stackrel{?}{=} 0$

$\vec{r} = (2-6, 0-2, -4+5)$

$= (-4,-2,1)$

$= \text{sqrt}(21)$

$\|r\| = \text{sqrt}(16+4+1)$

$\vec{v} = (2-1, 0+3, -4+2) = (1,3,-2)$ $\quad \|\vec{v}\|$

$\vec{r} = (-4,-2,1)$ $\quad \|w\|$

γ: $\vec{v} \cdot \vec{r} = -4 - 6 - 2 = -12$

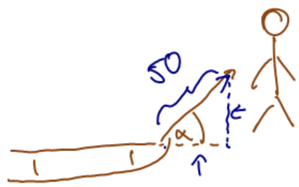
6

Panel 7

#29, p841

Force 50 N

$\alpha = 38^\circ$



50 sin(38)

50 cos(38)

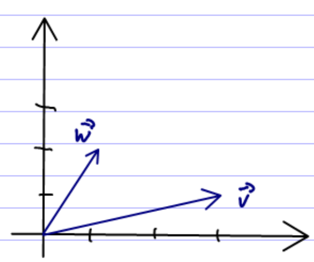
7

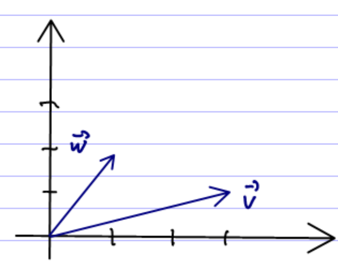
Panel 8

Name:

Quiz #2

(1) Suppose two vectors are drawn as indicated.





a) Draw $\vec{v} + \vec{w}$

b) Draw $\vec{v} - \vec{w}$

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Panel 9

Quiz #2

(2) If $\vec{v} = \langle 1, 2, 3 \rangle$ and $\vec{w} = 2\hat{i} + \hat{j} - 2\hat{k}$

a) find $\|\vec{v}\|$

b) find $2\vec{v} + \vec{w}$

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Panel 10

Quiz #2

(3) Let $\vec{v} = \langle 1, -2, 3 \rangle$ and $\vec{w} = \langle 2, -1, 3 \rangle$

a) Find a unit vector in the direction of $\vec{v}$

b) Find $\vec{v} \cdot \vec{w}$

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Panel 11

Quiz #2

(4) Compute the angle between $\langle 1,0,2\rangle$ and $\langle 0,3,\sqrt{15}\rangle$

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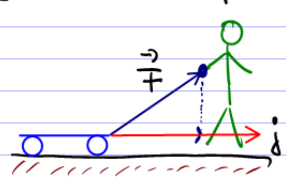
Panel 12

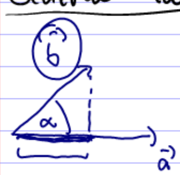
Prev. example: given 2 vectors, find resultant (no dot product)

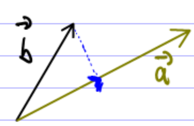
Now: How much of a given vector $\vec{v}$ goes in direction $\vec{w}$.

Ex:



General Question:





What is projection of $\vec{b}$ onto $\vec{a}$

how much of b goes in $\vec{a}$-direction

$$\text{proj}_{\vec{a}}\vec{b} = \underbrace{\frac{a\cdot b}{\|a\|}}_{\text{length}} \cdot \underbrace{\frac{\vec{a}}{\|a\|}}_{\text{direct.}} = \frac{a\cdot b}{\|a\|^2}\,\vec{a}$$

$\|b\|\cos(\alpha) =$

$$\|\cancel{b}\| \cdot \frac{a\cdot b}{\|a\|\,\|\cancel{b}\|} = \overbrace{\frac{a\cdot b}{\|a\|}}^{\text{scalar}} \cdot \frac{\vec{a}}{\|a\|}$$

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Panel 13

Projection Formula: $\text{proj}_{\vec a}\vec b = \frac{\vec a\cdot\vec b}{\|\vec a\|^2}\vec a = \frac{a\cdot b}{\|a\|}\cdot\frac{a}{\|a\|}$

Ex: Find length and direction of projection of $\vec b = \langle 1,1,2\rangle$ onto $\vec a = \langle -2,3,1\rangle$

Length: $3/\sqrt{14}$

dir: $\frac{1}{\sqrt{14}}\cdot\langle -2,3,1\rangle$

$$\text{proj}_a(b) = \frac{a\cdot b}{\|a\|^2}\cdot\vec a =$$

$$= \frac{\langle -2,3,1\rangle\cdot\langle 1,1,2\rangle}{\|\langle -2,3,1\rangle\|^2}\cdot\langle -2,3,1\rangle =$$

$$= \frac{-2+3+2}{(\sqrt{14})^2}\cdot\langle -2,3,1\rangle = \frac{3}{14}\langle -2,3,1\rangle$$

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Panel 14

More Multiplication of Vectors: Cross Product

Dot product of 2 vectors gives: number (scalar)

Cross product of 2 vectors gives vector

Def: If $\vec a = \langle a_1, a_2, a_3\rangle$ and $\vec b = \langle b_1, b_2, b_3\rangle$

then $\vec a\times\vec b = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1\rangle$

How on earth to memorize this?

Trick:

$$\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i\,(a_2b_3 - a_3b_2) - j\,(a_1b_3 - a_3b_1) + k\,(a_1b_2 - a_2b_1)$$

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Panel 15

Example: $\langle 1,3,4\rangle \underset{\text{cross-product}}{\times} \langle 2,7,-5\rangle = \langle -15-28, -(-5-8), 7-6 \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix} = i\,(3\cdot(-5)-7\cdot 4) - j\,(1(-5)-2\cdot 8) + k\,(1\cdot 7-2\cdot 3) =$$

Ex: $\langle 1,0,3\rangle \times \langle 2,3,-1\rangle = \langle -9, 7, 3\rangle = -9i+7j+3k$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 3 & -1 \end{vmatrix} = \langle 0-9, -(-1-6), 3-0 \rangle$$

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Panel 16

Properties of Cross Product

a) $\vec{a} \times \vec{a} = 0$

b) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

c) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

~~d) $a \times (b \times c) = (a\cdot c)\, b - (a \cdot b)\, c$~~

Proof a) $a = \langle a_1, a_2, a_3 \rangle$ $\quad a \times a = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = \langle a_2 a_3 - a_3 a_2, 0, 0 \rangle$

Proof b) HW DO IT

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Panel 17

Dot vs Cross Product

dot product gives scalar

$$\frac{a \cdot b}{\|a\| \|b\|} = \cos(\alpha)$$

cross prod gives vector

$$\frac{\|a \times b\|}{\|a\| \|b\|} = \sin(\alpha)$$

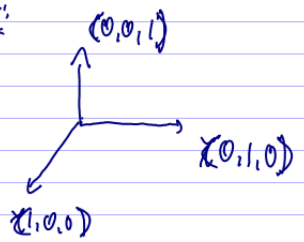
dot prod. = 0

$\Rightarrow$ vectors are perp.

(cross prod = 0

$\Rightarrow$ vectors parallel)

Ex:



$\Leftarrow$ cross product of $\vec{a}$ and $\vec{b}$ is a vector perp. to both, i.e. to plane spanned by $\vec{a}$, $\vec{b}$

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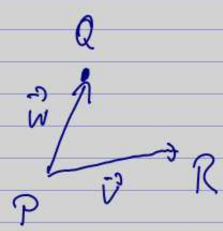
Panel 18

Example: Find vector perp. to plane through $P(1,4,6)$, $Q(-2,5,-1)$ and $R(1,-1,1)$



$\vec{v} = \langle 0, -5, -5 \rangle = R - P$

$\vec{w} = \langle -3, 1, -7 \rangle = Q - P$

$$\begin{vmatrix} i & j & k \\ 0 & -5 & -5 \\ -3 & 1 & -7 \end{vmatrix} = \langle 35+5, -15, 15 \rangle =$$

$$= \underline{\langle 40, -15, 15 \rangle} = \underline{5 \langle 8, -3, 3 \rangle}$$

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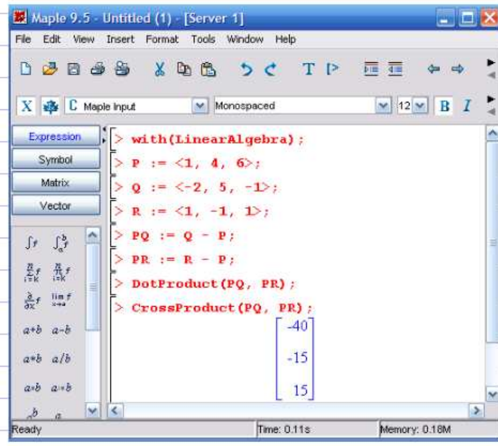
Panel 19

Maple: Dot + Cross Product



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Panel 20

Pop Quiz

Use Maple to find

① $\langle 1,3,9\rangle \cdot \langle 7,-2,-5\rangle$

a) 0 c) -44

b) 44 d) 12

② $\langle 1,3,9\rangle \times \langle 7,-2,-5\rangle$

a) 0 b) $\langle 3,-68,-23\rangle$

c) $\langle 1,2,3\rangle$ d) $\langle 3,68,-23\rangle$

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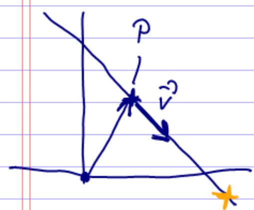
Panel 21

Lines in 3D:

Know lines in 2D: $y = \underbrace{mx}_{\text{slope}} + \underbrace{b}_{\text{intercept}}$, $z=0$

Not good: can't easily extend to $\mathbb{R}^3$; not all lines work! (vert.)



to reach any point on this line go to P, then add $t \cdot \vec{v}$

$\Rightarrow \ell(t) = P + t \cdot \vec{v}$

P is point on line
$\vec{v}$ is direction of line

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Panel 22

A line through $r_0 = (x_0, y_0, z_0)$ in direction of $\vec{v} = \langle a, b, c\rangle$ is $\ell(t) = r_0 + t\vec{v} = \langle x_0 + ta, y_0 + tb, z_0 + tc\rangle$

Ex: Find line through $(5,1,3)$ parallel to $\langle 1,4,-2\rangle$

$r_0 = (5,1,3)$
$\vec{v} = \langle 1,4,-2\rangle$ $\Rightarrow \ell(t) = \langle 5+t, 1+4t, 3-2t\rangle$

Ex: Find line through $(1,2,3)^P$ and $(3,2,1)^Q$

$r_0 = (1,2,3)$
$\vec{v} = \vec{PQ} = \langle 3-1, 2-2, 1-3\rangle = \langle 2,0,-2\rangle$

$\ell(t) = \langle 1+2t, 2, 3-2t\rangle$

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Panel 23

In $\mathbb{R}^2$: line through $(1,2)$ and $(4,3)$

Yesterday (OLD): $y = mx + b = \frac{3-2}{4-1}x + b = \frac{1}{3}x + b$

$2 = \frac{1}{3} 1 + b \Rightarrow b = 5/3$

$\Rightarrow y = \frac{1}{3}x + 5/3$

NEW: $l(t) = \langle 1 + t \cdot 3, 2 + t \cdot 1 \rangle =$

$= \langle 1 + 3t, 2 + t \rangle$

means: $x = 1 + 3t \Rightarrow x - 1 = 3t \quad \frac{1}{3}x - \frac{1}{3} = t$

$y = 2 + t$

$, y = 2 + \frac{1}{3}x - \frac{1}{3} = 5/3 + \frac{1}{3}x$

through $(1,3), (1,5)$

OLD: $m = 2/0$ undef

$x = 1$

NEW:

$l(t) = \langle 1 + t0, 3 + t2 \rangle$

$= \langle 1, 3 + 2t \rangle$

Panel 24

Ex: Suppose 2 lines are $l_1(t) = \langle 1+t, -2+3t, 4-t \rangle$

$l_2(s) = \langle 2s, 3+s, -3+4s \rangle$

a) Are the lines parallel?

l_1 has direction $\langle 1, 3, -1 \rangle$

l_2 has direction $\langle 2, 1, 4 \rangle$

No (vectors are not parallel) (mult. of each other)

b) Do they intersect in $\mathbb{R}^3$?

If they did intersect: $l_1(t) = l_2(s)$

$1 + t = 2s \Rightarrow t = 2s - 1$

$-2 + 3t = 3 + s \quad -2 + 3(2s-1) = 3 + s \Rightarrow -2 + 6s - 3 = 3 + s, \; s = 8/5$

$t = 11/5$

$(4 - t = -3 + 4s)$ check in this equation: false

Panel 25

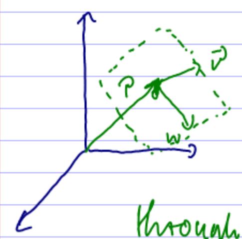
Planes in $\mathbb{R}^3$

Line determined by: 2 points or 1 point + dir vector

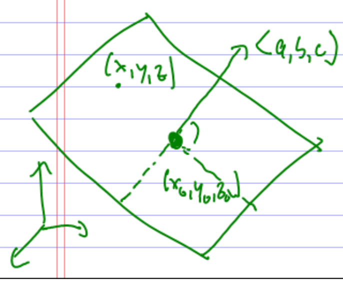
Plane determined by: 3 points or

1 point + 2 vectors

or 1 point + 1 perp. vector



Suppose (x, y, z) is on the plane through (x_0, y_0, z_0) perp. to $\langle a, b, c\rangle$



$\Rightarrow PX = \langle x-x_0, y-y_0, z-z_0\rangle \cdot \langle a,b,c\rangle = 0$

$a(x-x_0)+b(y-y_0)+c(z-z_0) = 0$

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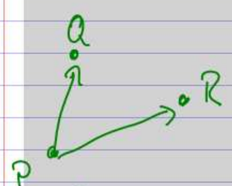
Panel 26

Scalar equation of Plane through $P(x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c\rangle$ is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Ex: Plane through $P(1,3,2)$, $Q(3,-1,6)$, and $R(5,2,0)$



equation of plane:

need: point : $P(1,3,2)$

normal (perp.) vector

$\vec{PR} = \langle$

$\vec{PQ} =$

$\vec{PR} \times \vec{PQ} = \langle \overset{a}{5}, \overset{b}{-1}, \overset{c}{9}\rangle$?

$5(x-1)-1(y-3)+9(z-2)=0$

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