Panel 1

Math 2411 - Calc 3

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This class will use DyKnow Collaboration software to test collaborative work in a math lecture!

⇒ Hardware: Your laptop (Tablet preferred) Bring to class

Software: DyKnow (Download + install)

Homepage for this class:

http://pirate.shu.edu/~wachsmut/

1

Panel 2

About this Class

Lectures + HW - no points

Quizzes: 100 points total (2 worst dropped)

3 exams: 300 P

(1 final. 100 P)

Maple 100 P

500 / 600 → final grade

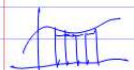
2

Panel 3

Now it's time for some mathematics

Panel 4

Calc I – Overview

	Def. ✓	Geometry ✓	How-to ✓
• Limit	Given $\varepsilon > 0$ there is $\delta > 0$ such that if $\lvert x-c \rvert < \delta$ then $\lvert f(x) - L \rvert < \varepsilon$	as x gets close to c, $f(x)$ gets close to limit L	plug in value, hope for the best; l'Hospital
• Continuity	$\lim_{x \to c} f(x) = f(c)$	 no gaps, holes	see limits
• Differentiation	$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$	slope of tangent, rate of change	Power Rule, Quotient, Chain, Product
• Integration 	$\lim_{\lVert \Delta x \rVert \to 0} \sum_{i=1}^{n} f(x_i) \Delta x_i = \int_a^b f(x)\,dx$	area under curve, (if $f(x) \geq 0$)	antideriv, Fund. Thm.

Panel 5

Ex:

$$\lim_{x\to 3} \frac{x^2-9}{x} = 0$$

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = 6$$

$$\lim_{x\to 0} \frac{\sin(2x)}{x} = \lim_{x\to 0} \frac{2\cos(2x)}{1} = 2$$

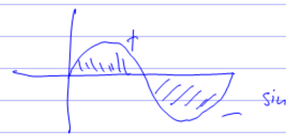
$[(x^2-1)^{1/2}]'$ ↓

$$f(x) = \frac{e^x \cdot \ln(x)}{\sqrt{x^2-1}}, \quad f'(x) = \frac{(e^x \ln(x) + \frac{e^x}{x})\sqrt{x^2-1} - e^x \ln(x)\left[\frac{1}{2}(x^2-1)^{-1/2} 2x\right]}{(\sqrt{x^2-1})^2}$$

$$\int_0^{2\pi} \sin(x)\,dx = -\cos(x)\Big|_0^{2\pi} = -\cos(2\pi) + \cos(0) = 0$$

↑ antideriv.

Fund. theorem of Calc



Panel 6

Calc 2 - Overview

- Heavy Duty Integration Techniques
  - e.g.: by parts, partial fract. decomp, trig subst.
- Applications of Integration
  - e.g.: volume of rotating curves, lengths, center of mass, work
- Sequences & Series
  - e.g.: geom $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, if $|x|<1$, e^x, Ratio test & Div. test
- Differential Equations
  - e.g. 1^{st} order linear, separable, (exact)

Panel 7

Ex: $\int \frac{1}{x(x+2)}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{1}{x+2}\,dx = \frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$

$\int_0^{\pi} x\sin(x)\,dx = -x\cos(x)\Big|_0^{\pi} + \int_0^{\pi}\cos(x)\,dx =$

$u = x$, $u' = 1$

$v' = \sin(x)$, $v = -\cos(x)$

$= -\pi\cos(\pi) + 0\cdot\cos 0 + \sin(x)\Big|_0^{\pi} =$

$= \pi + \sin(\pi) - \sin(0) = \pi$

$\frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} = \frac{A(x+2)+Bx}{x(x+2)} \Rightarrow$

$A(x+2) + Bx = 1$

$x = -2: \quad -2B = 1 \Rightarrow B = -1/2$

$x = 0: \quad 2A = 1 \Rightarrow A = 1/2$

Panel 8

Find series of $\frac{1}{1+2x^2} = \frac{1}{1-(-2x^2)} = \sum_{n=0}^{\infty}(-2x^2)^n = \sum_{n=0}^{\infty}(-2)^n x^{2n}$

Know $\frac{1}{1-\bigcirc} = \sum_{n=0}^{\infty}\bigcirc^n$

$xy' = e^{-y}\ln(x)$ Find y'

$y'e^y = \frac{1}{x}\ln(x)$, $e^y\frac{dy}{dx} = \frac{1}{x}\ln(x)$

$\Rightarrow e^y\,dy = \frac{1}{x}\ln(x)\,dx$ then integrate

$\int e^y\,dy = \int\left(\frac{1}{x}\right)\ln(x)\,(dx)$ $\quad u = \ln(x),\ du = \frac{1}{x}dx \quad = \int u\,du$

$e^y = \frac{1}{2}(\ln(x))^2 + C \Rightarrow y = \ln\left(\frac{1}{2}(\ln(x))^2 + C\right)$

Panel 9

Calc 3

Calc 1 + Calc 2 + multiple variables = Calc 3

$\mathbb{R}^3$ (vectors, adding, "multiply" ...)

$f: \mathbb{R} \to \mathbb{R}$

Vector valued functions → $f: \mathbb{R} \to \mathbb{R}^2$ (1)

⇒ (lim, cont, diff, int)

Functions of 2 variables → $f: \mathbb{R}^2 \to \mathbb{R}$ (2)

⇒ (lim, cont., diff, int, max/min)

Multiple Integral + Appl.

Icing on the cake

⇒ (Vector fields) → $f: \mathbb{R}^2 \to \mathbb{R}^2$ (3)

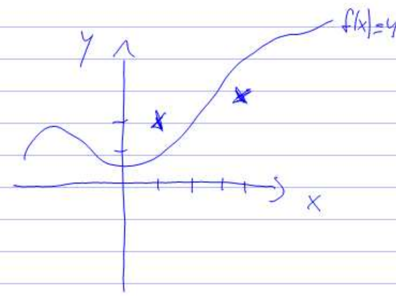
9

Panel 10

Introducing $\mathbb{R}^3$

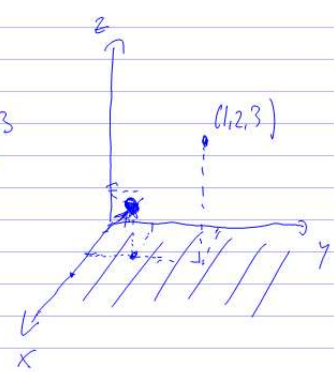
Coordinate system in $\mathbb{R}^2$:

Ex: plot (1,2), (4,3)



Coordinate system in $\mathbb{R}^3$:

Ex: plot (1,1,1), (1,2,3)



x-axis: all points (t, 0, 0) ← $f: \mathbb{R} \to \mathbb{R}^3$

y-axis: (0, t, 0)

z-axis: (0, 0, t)

xy plane: all points (t, s, 0)

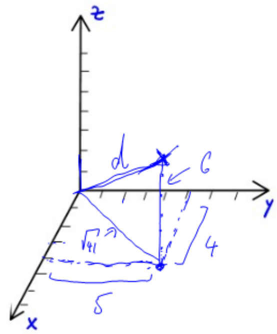
yz plane: (0, s, t)

xz plane: (s, 0, t)

10

Panel 11

Ex: Plot the following Points:



$P(3,2,1)$ - use blue

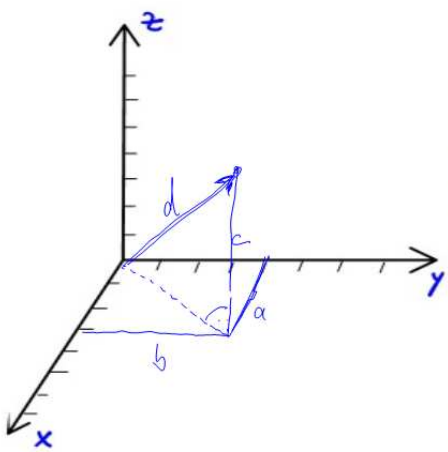
$Q(4,5,6)$ - use green

length $(0,0,0)$ to Q: $d = \sqrt{41+36} = \sqrt{5^2+4^2+6^2}$

11

Panel 12

Distance in R^3



$P(a,b,c)$

Distance between origin and P

$d^2 = a^2+b^2+c^2$

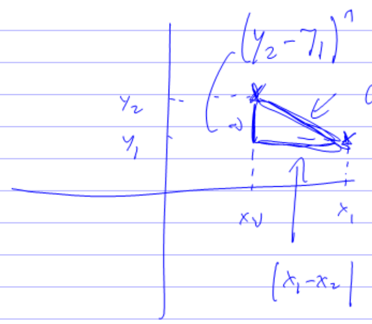
$\Rightarrow d = \sqrt{a^2+b^2+c^2}$

Distance between $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$:

$$d_{\vec{PQ}} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

12

Panel 13



$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Ex: Find distance between $(1,2,3)$ and $(3,2,1)$

$d=\sqrt{(3-1)^2+(2-2)^2+(1-3)^2} = \sqrt{8}$

$\quad 4 \quad 0 \quad 4$

13

Panel 14

3D Objects

$P(x,y,z) \Rightarrow d=\sqrt{x^2+y^2+z^2} \Rightarrow d^2 = x^2+y^2+z^2$

All points with fixed distance to origin in 3D is a SPHERE

Def: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$ is a sphere with radius r and center (x_0,y_0,z_0)

Ex: Find radius and center of sphere

$x^2+y^2+z^2-2x-4y+8z+17=0$

center $(1,2,-4)$
radius 2

$x^2-2x+y^2-4y+z^2+8z=-17$

$(x-1)^2-1+(y-2)^2-4+(z+4)^2-16=-17$

$(x^2-2x+1 \quad y^2-4y+4 \quad (z^2+8z+16)$

$(x-1)^2+(y-2)^2+(z+4)^2 = -17+1+4+16=4$

14

Panel 15

Ex: Find the center + radius of the sphere given by:

$$x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 = 0$$

center: $(-5, -2, -1)$

radius: $\sqrt{49} = 7$

$$(x+5)^2 + (y+2)^2 + (z+1)^2 = 19 + 25 + 4 + 1$$
$$= 49$$

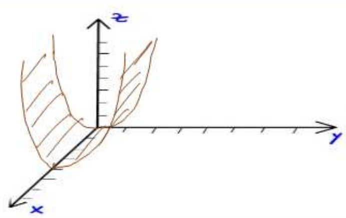
15

Panel 16

Objects with 2 Variables in 3D

Graph of object with 2 variables in 3D space is a "sheet":

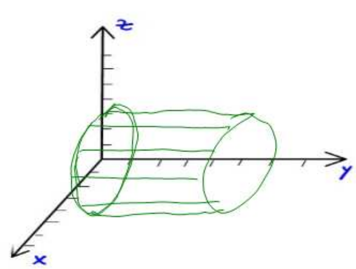
Ex: $z = y^2$



Sheets

(one free variable)

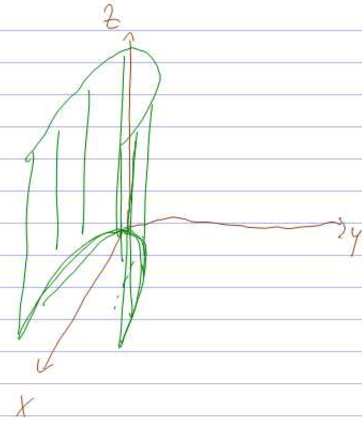
Ex: $x^2 + z^2 = 1$



16

Panel 17

$z = y^2$



17

Panel 18

Drawing 3D objects with Maple

Maple can easily draw 3D objects

```
> with(plots);
> implicitplot3d(z=y^2, x=-3..3, y=-3..3, z=-1..9);
> plot3d(x^2, x=-3..3, y=-3..3);
> implicitplot3d(z^2+y^2=4, x=-3..3, y=-3..3, z=-3..3);
```

18

Panel 19

Ex: Use Maple to graph the following and insert them here via copy + paste.

$x^2+y^2+z^2=4$ ← sphere, (0,0,0), r = 2

$y^2+z^2=2$ cyl. around x-axis

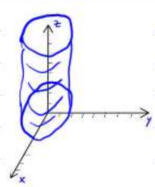
$z=\sin(x)\cdot\cos(y)$ ← unknown to us

19

Panel 20

Quick Quiz

(1)

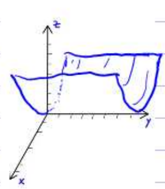


a) $x^2+y^2=1$

b) $x^2+z^2=1$

c) $y^2+z^2=1$

(2)

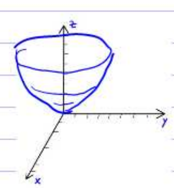


a) $y=x^2$

b) $z=x^2$

c) $z=y^2$

(3)



a) $z=xy$

b) $z=x^2-y^2$

c) $z=x^2+y^2$

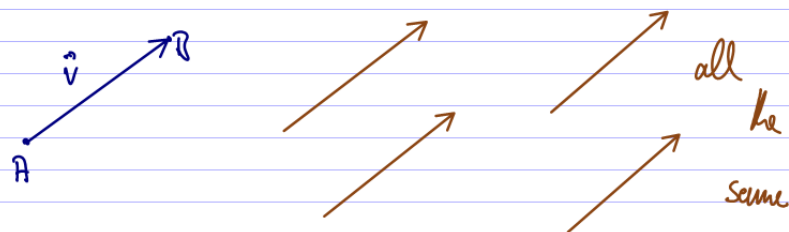
20

Panel 21

Vectors

Understand points in 3D (and 2D). Want to investigate more general objects ⇒ vectors.

Def: A vector is a directed line segment, i.e. part of a line with a direction and a length. A vector from point A to B is written as $\vec{v} = \vec{AB}$ and is visualized as arrow from A to B.



21

Panel 22

Vector Math, ~~Geometrically~~ Algebraically

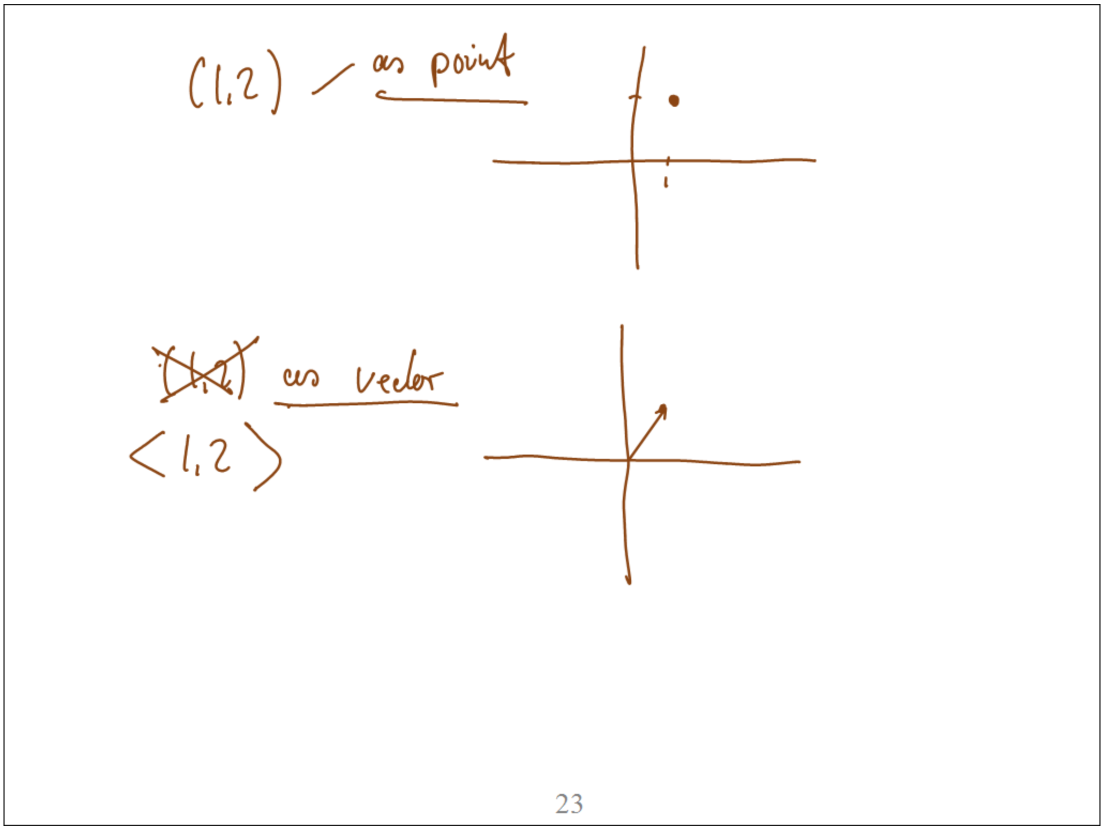
If $\vec{v}$ is a vector then $k \cdot v$ is k-times as long — $\vec{v} = \langle 1,2 \rangle$, $3 \cdot \vec{v} = \langle 3,6 \rangle$

If v, w are vectors, then v+w is another vector — $\langle 1,2 \rangle + \langle 3,4 \rangle = \langle 4,6 \rangle$

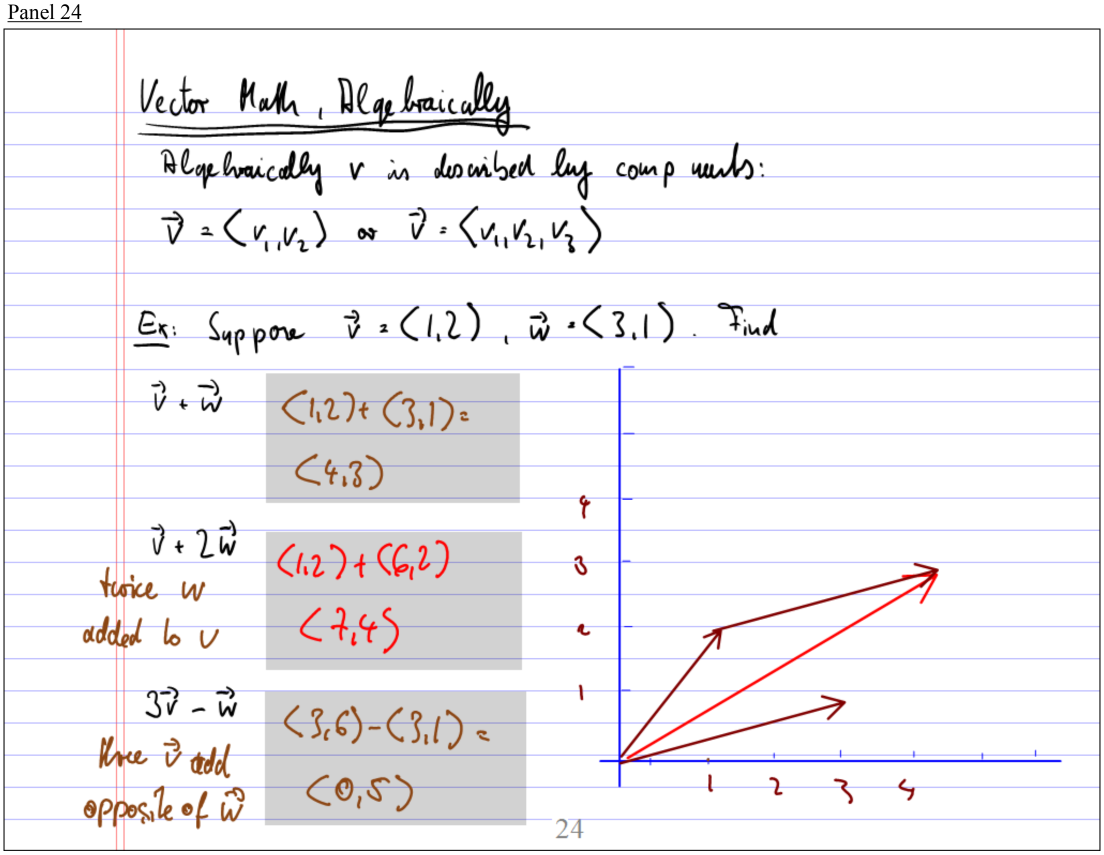
If v, w are vectors, then v-w is another vector — $\langle 1,2 \rangle - \langle 3,4 \rangle = \langle -2,-2 \rangle$

22

Panel 23



Panel 24



Panel 25