

Panel 1

Last Time

Parametric curves: $(x,y) = (f(t), g(t))$ ↙ base direction and speed

Example: $(t+3, -2t+1)$ line with y-intercept $(0, 7)$

$(t^2+t, t+5)$ $x=t+1 \Rightarrow x-1=t, y=t^2+3 = (x-1)^2+3$

Must know: $(3 \cos(t), 3 \sin(t))$ circle, ellipse

$(\pm \cos(t), \pm \sin(t))$ spiral

$(3 \sin(2t), 3 \cos(2t))$

Don't forget Ovalbums $(\cos(2t), \sin(3t))$

Panel 2

different periods give Lissajous Figures

`plot([cos(3 t), sin(7 t), t=0..7])`

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Panel 3

Parametric curves can be very pretty:

Plot $(t + 2 \sin(2t), t + 2 \cos(5t))$

`plot([t + 2 sin(2 t), t + 2 cos(5 t), t=0..7])`

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Panel 4

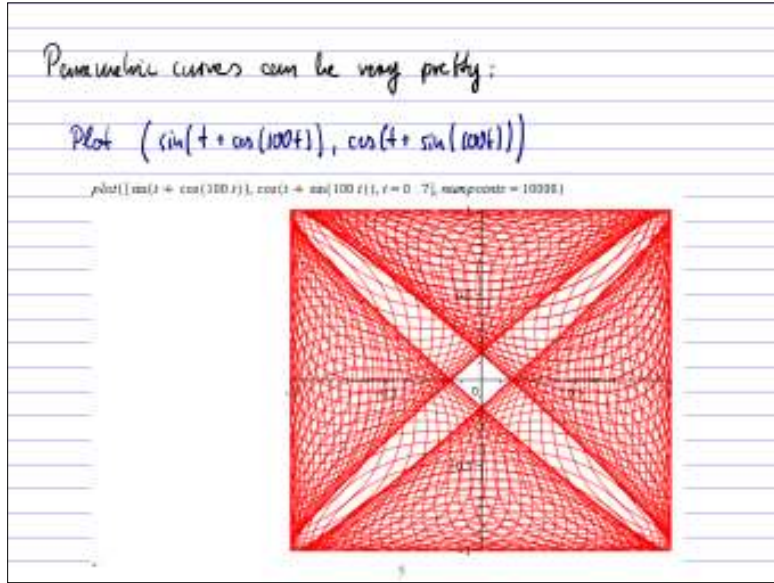
Parametric curves can be very pretty:

Plot $(2 \cos(t) - \cos(30t), 2 \sin(t) - \sin(30t))$

`plot([2 cos(t) - cos(30 t), 2 sin(t) - sin(30 t), t=0..7])`

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Panel 5



Panel 6

Def. A parametric curve $(x(t), y(t))$ is called smooth if $x'(t)$ and $y'(t)$ are not simultaneously zero.

Ex: Which curve is smooth, which one not?

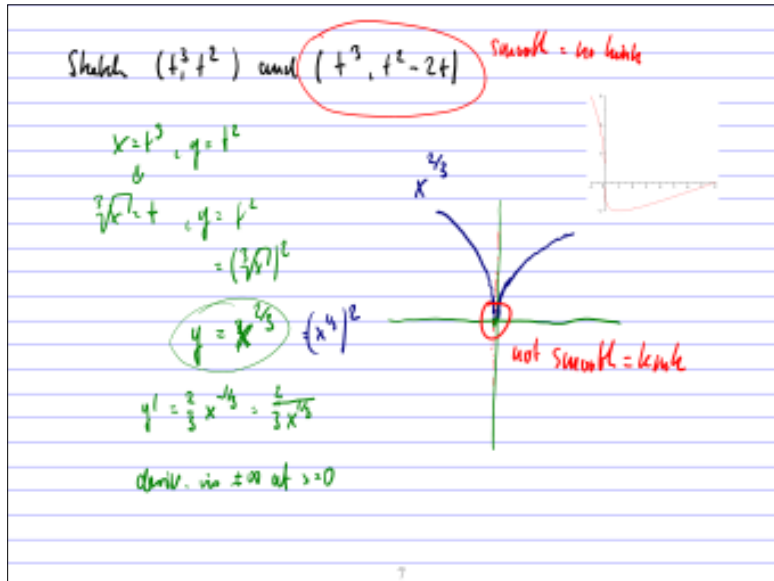
a) (t^3, t^2) b) $(t^3, t^2 - 2t)$

$x(t) = t^3 \Rightarrow x'(t) = 3t^2$ $x(t) = t^3, x'(t) = 3t^2 = 0, t = 0$

$y(t) = t^2 \Rightarrow y'(t) = 2t = 0$ $y(t) = t^2 - 2t, y'(t) = 2t - 2 = 0, t = 1$

at $t=0: x'(t) = y'(t) = 0 \Rightarrow$ no smooth smooth

Panel 7



Panel 8

Theorem: If $(x(t), y(t))$ is a smooth parametric curve, then the slope ^{at limit} is:

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$$

Ex: slope of $(2t, 6t)$ $\Rightarrow \frac{y'}{x'} = 6t = 3(2t) = 3x$

slope, $\frac{y'(t)}{x'(t)} = \frac{6}{2} = 3$

Panel 9

Find slope of $(\sqrt{t}, \frac{1}{4}(t^2-4))$ at $(1, -\frac{3}{4})$

$$x'(t) = \frac{1}{2\sqrt{t}}, \quad y'(t) = \frac{1}{2} \cdot 2t = \underline{\underline{\frac{1}{2}t}}$$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\frac{1}{2}t}{\frac{1}{2\sqrt{t}}} = t\sqrt{t} = \underline{\underline{\frac{3\sqrt{2}}{2}}}$$

what is t so that $(\sqrt{t}, \frac{1}{4}(t^2-4)) = (1, -\frac{3}{4}) \Rightarrow t=1$

Panel 10

Find slope of a circle of radius 1 at angle $\frac{\pi}{4}$

① $y = \sqrt{1-x^2}, \quad y'(x) = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot (-2x) = \frac{-x}{\sqrt{1-x^2}} = \underline{\underline{-\frac{1}{\sqrt{2}}}}$

② $(\cos(t), \sin(t))$ at $\frac{\pi}{4}$ in angle

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-\sin(t)}{\cos(t)} \Big|_{t=\frac{\pi}{4}} = \frac{-\sin(\frac{\pi}{4})}{\cos(\frac{\pi}{4})} = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \underline{\underline{-1}}$$

Panel 11

Quiz on Monday (last quiz) on parametric equation:

- ① What's the graph? linear
circle, ellipse, spiral
- ② Write parametric as $y = \dots$ e.g. $(t-1, t^2+2)$
i.e. eliminate the t . $x=t+1 \Rightarrow t=x-1$
 $y = (x-1)^2 + 2$
- ③ Is it smooth?
- ④ Find slope of tangent!

Panel 12

Ex: Consider (t^2, t^3-3t)

a) Is it smooth?

$$x'(t) = 2t = 0 \Rightarrow t=0$$

$$y'(t) = 3t^2 - 3 = 0$$

smooth

b) Show that the curve has two tangents at $(3,0)$ and find their equation.

$t^2 = 3 \Rightarrow t = \pm\sqrt{3}$ Here are 2 values for which curve passes through $(3,0)$

$$y(\pm\sqrt{3}) = \begin{cases} 9\sqrt{3} - 3\sqrt{3} = 0 \\ -9\sqrt{3} + 3\sqrt{3} = 0 \end{cases}$$

slope: $\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3t^2-3}{2t}$

$t = \sqrt{3} : \frac{6-3}{2\sqrt{3}} = \frac{3}{2\sqrt{3}}$

$t = -\sqrt{3} : \frac{6-3}{-2\sqrt{3}} = -\frac{3}{2\sqrt{3}}$

Panel 13

$$(t^2, t^3 - 3t) = (3, 0) \text{ for } t = -\sqrt{3}, t = \sqrt{3}$$

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Panel 14

Ex: $(t^2, t^3 - 3t)$ - find horizontal/vertical tangents.

$$\frac{dy}{dx} = \frac{3t^2 - 3}{2t} \text{ is horizontal if } t = \pm 1 : (1, -2) \text{ } t = -1$$

$$\text{is vertical if } t = 0 : (0, 0)$$

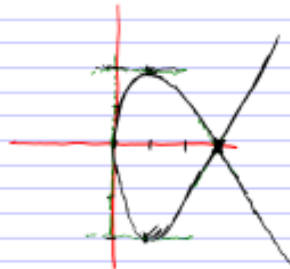
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Panel 15

Ex: $(t^2, t^3 - 3t)$ - sketch curve

Know:

- passes through $(0, 0)$
twice, once with pos slope, once neg.
- horiz. tangent at $(1, 2), (1, -2)$
- vertical tangent at $(0, 0)$



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