

where a represents the overhead cost (rent, heat, maintenance) and the other terms represent the cost of raw materials, labor, and so on. (The cost of raw materials may be proportional to x , but labor costs might depend partly on higher powers of x because of overtime costs and inefficiencies involved in large-scale operations.)

For instance, suppose a company has estimated that the cost (in dollars) of producing x items is

$$C(x) = 10,000 + 5x + 0.01x^2$$

Then the marginal cost function is

$$C'(x) = 5 + 0.02x$$

The marginal cost at the production level of 500 items is

$$C'(500) = 5 + 0.02(500) = \$15/\text{item}$$

This gives the rate at which costs are increasing with respect to the production level when $x = 500$ and predicts the cost of the 501st item.

The actual cost of producing the 501st item is

$$\begin{aligned} C(501) - C(500) &= [10,000 + 5(501) + 0.01(501)^2] \\ &\quad - [10,000 + 5(500) + 0.01(500)^2] \\ &= \$15.01 \end{aligned}$$

Notice that $C'(500) \approx C(501) - C(500)$. ■

2.3 EXERCISES

1–24 ■ Differentiate the function.

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| 1. $f(x) = 186.5$ | 2. $f(x) = \sqrt{30}$ |
| 3. $f(x) = 5x - 1$ | 4. $F(x) = -4x^{10}$ |
| 5. $f(x) = x^3 - 4x + 6$ | 6. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$ |
| 7. $f(x) = x - 3 \sin x$ | 8. $y = \sin t + \pi \cos t$ |
| 9. $f(t) = \frac{1}{4}(t^4 + 8)$ | 10. $h(x) = (x - 2)(2x + 3)$ |
| 11. $y = x^{-2/5}$ | 12. $R(t) = 5t^{-3/5}$ |
| 13. $V(r) = \frac{4}{3}\pi r^3$ | 14. $R(x) = \frac{\sqrt{10}}{x^7}$ |
| 15. $F(x) = \left(\frac{1}{2}x\right)^5$ | 16. $y = \sqrt{x}(x - 1)$ |
| 17. $y = 4\pi^2$ | 18. $g(u) = \sqrt{2}u + \sqrt{3u}$ |
| 19. $y = \frac{x^2 + 4x + 3}{\sqrt{x}}$ | 20. $y = \frac{x^2 - 2\sqrt{x}}{x}$ |
| 21. $v = t^2 - \frac{1}{\sqrt[3]{t^3}}$ | 22. $y = \frac{\sin \theta}{2} + \frac{c}{\theta}$ |
| 23. $z = \frac{A}{y^{10}} + B \cos y$ | 24. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$ |

25–26 ■ Find equations of the tangent line and normal line to the curve at the given point.

25. $y = 6 \cos x, (\pi/3, 3)$ 26. $y = (1 + 2x)^2, (1, 9)$

27–28 ■ Find an equation of the tangent line to the curve at the given point. Illustrate by graphing the curve and the tangent line on the same screen.

27. $y = x + \sqrt{x}, (1, 2)$ 28. $y = 3x^2 - x^3, (1, 2)$

29–32 ■ Find the first and second derivatives of the function.

29. $f(x) = x^4 - 3x^3 + 16x$ 30. $G(r) = \sqrt{r} + \sqrt[3]{r}$
 31. $g(t) = 2 \cos t - 3 \sin t$ 32. $h(t) = \sqrt{t} + 5 \sin t$

33. Find $\frac{d^{99}}{dx^{99}}(\sin x)$.

34. Find the n th derivative of each function by calculating the first few derivatives and observing the pattern that occurs.
 (a) $f(x) = x^n$ (b) $f(x) = 1/x$