where a represents the overhead cost (rent, heat, maintenance) and the other terms represent the cost of raw materials, labor, and so on. (The cost of raw materials may be proportional to x, but labor costs might depend partly on higher powers of x because of overtime costs and inefficiencies involved in large-scale operations.)

For instance, suppose a company has estimated that the cost (in dollars) of producing x items is

$$C(x) = 10{,}000 + 5x + 0.01x^2$$

Then the marginal cost function is

$$C'(x) = 5 + 0.02x$$

The marginal cost at the production level of 500 items is

$$C'(500) = 5 + 0.02(500) = \$15/\text{item}$$

This gives the rate at which costs are increasing with respect to the production level when $x = 500$ and predicts the cost of the 501st item.

The actual cost of producing the 501st item is

$$\begin{aligned} C(501) - C(500) &= [10{,}000 + 5(501) + 0.01(501)^2] \\ &\qquad - [10{,}000 + 5(500) + 0.01(500)^2] \\ &= \$15.01 \end{aligned}$$

Notice that $C'(500) \approx C(501) - C(500)$. ■

2.3 EXERCISES

1–24 ▪ Differentiate the function.

1. $f(x) = 186.5$

2. $f(x) = \sqrt{30}$

3. $f(x) = 5x - 1$

4. $F(x) = -4x^{10}$

5. $f(x) = x^3 - 4x + 6$

6. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$

7. $f(x) = x - 3\sin x$

8. $y = \sin t + \pi\cos t$

9. $f(t) = \frac{1}{4}(t^4 + 8)$

10. $h(x) = (x - 2)(2x + 3)$

11. $y = x^{-2/5}$

12. $R(t) = 5t^{-3/5}$

13. $V(r) = \frac{4}{3}\pi r^3$

14. $R(x) = \dfrac{\sqrt{10}}{x^7}$

15. $F(x) = (\frac{1}{2}x)^5$

16. $y = \sqrt{x}\,(x - 1)$

17. $y = 4\pi^2$

18. $g(u) = \sqrt{2}\,u + \sqrt{3u}$

19. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

20. $y = \dfrac{x^2 - 2\sqrt{x}}{x}$

21. $v = t^2 - \dfrac{1}{\sqrt[4]{t^3}}$

22. $y = \dfrac{\sin\theta}{2} + \dfrac{c}{\theta}$

23. $z = \dfrac{A}{y^{10}} + B\cos y$

24. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$

25–26 ▪ Find equations of the tangent line and normal line to the curve at the given point.

25. $y = 6\cos x, \quad (\pi/3, 3)$

26. $y = (1 + 2x)^2, \quad (1, 9)$

27–28 ▪ Find an equation of the tangent line to the curve at the given point. Illustrate by graphing the curve and the tangent line on the same screen.

27. $y = x + \sqrt{x}, \quad (1, 2)$

28. $y = 3x^2 - x^3, \quad (1, 2)$

29–32 ▪ Find the first and second derivatives of the function.

29. $f(x) = x^4 - 3x^3 + 16x$

30. $G(r) = \sqrt{r} + \sqrt[3]{r}$

31. $g(t) = 2\cos t - 3\sin t$

32. $h(t) = \sqrt{t} + 5\sin t$

33. Find $\dfrac{d^{99}}{dx^{99}}(\sin x)$.

34. Find the nth derivative of each function by calculating the first few derivatives and observing the pattern that occurs.
(a) $f(x) = x^n$ (b) $f(x) = 1/x$