

In the definition of an increasing function it is important to realize that the inequality $f(x_1) < f(x_2)$ must be satisfied for every pair of numbers x_1 and x_2 in I with $x_1 < x_2$.

You can see from Figure 18 that the function $f(x) = x^2$ is decreasing on the interval $(-\infty, 0]$ and increasing on the interval $[0, \infty)$.

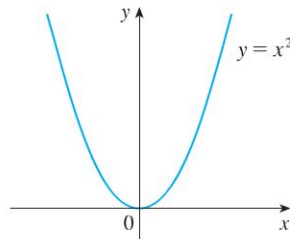
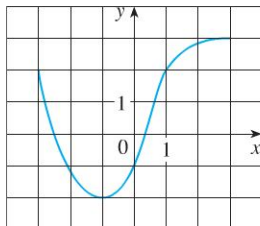


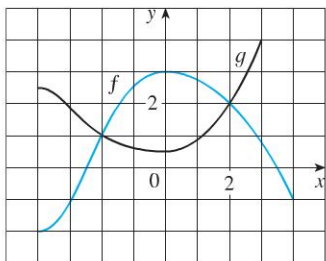
FIGURE 18

1.1 EXERCISES

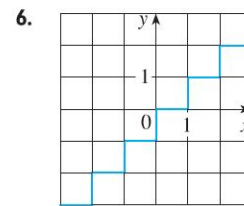
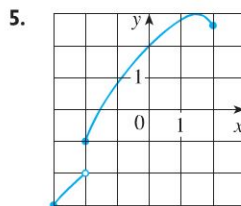
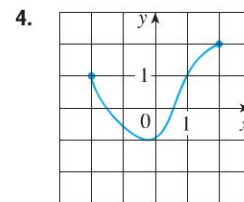
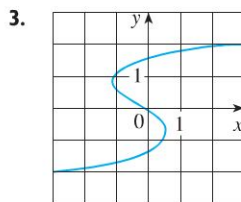
1. The graph of a function f is given.
 - (a) State the value of $f(-1)$.
 - (b) Estimate the value of $f(2)$.
 - (c) For what values of x is $f(x) = 2$?
 - (d) Estimate the values of x such that $f(x) = 0$.
 - (e) State the domain and range of f .
 - (f) On what interval is f increasing?



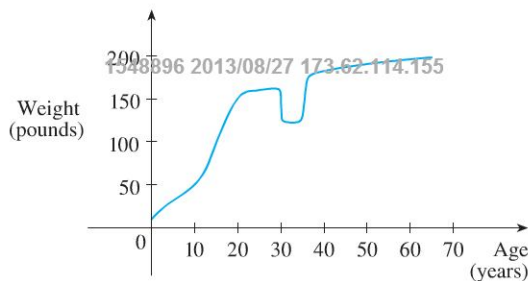
2. The graphs of f and g are given.
 - (a) State the values of $f(-4)$ and $g(3)$.
 - (b) For what values of x is $f(x) = g(x)$?
 - (c) Estimate the solution of the equation $f(x) = -1$.
 - (d) On what interval is f decreasing?
 - (e) State the domain and range of f .
 - (f) State the domain and range of g .



- 3–6 ■ Determine whether the curve is the graph of a function of x . If it is, state the domain and range of the function.

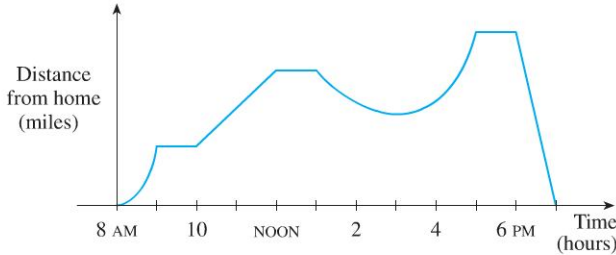


7. The graph shown gives the weight of a certain person as a function of age. Describe in words how this person's weight varies over time. What do you think happened when this person was 30 years old?



SECTION 1.1 FUNCTIONS AND THEIR REPRESENTATIONS ■ 9

8. The graph shown gives a salesman's distance from his home as a function of time on a certain day. Describe in words what the graph indicates about his travels on this day.



9. You put some ice cubes in a glass, fill the glass with cold water, and then let the glass sit on a table. Describe how the temperature of the water changes as time passes. Then sketch a rough graph of the temperature of the water as a function of the elapsed time.
10. Sketch a rough graph of the number of hours of daylight as a function of the time of year.
11. Sketch a rough graph of the outdoor temperature as a function of time during a typical spring day.
12. Sketch a rough graph of the market value of a new car as a function of time for a period of 20 years. Assume the car is well maintained.
13. Sketch the graph of the amount of a particular brand of coffee sold by a store as a function of the price of the coffee.
14. You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool before eating it. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.
15. A homeowner mows the lawn every Wednesday afternoon. Sketch a rough graph of the height of the grass as a function of time over the course of a four-week period.
16. A jet takes off from an airport and lands an hour later at another airport, 400 miles away. If t represents the time in minutes since the plane has left the terminal, let $x(t)$ be the horizontal distance traveled and $y(t)$ be the altitude of the plane.
- Sketch a possible graph of $x(t)$.
 - Sketch a possible graph of $y(t)$.
 - Sketch a possible graph of the ground speed.
 - Sketch a possible graph of the vertical velocity.
17. If $f(x) = 3x^2 - x + 2$, find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$, $f(a + 1)$, $2f(a)$, $f(2a)$, $f(a^2)$, $[f(a)]^2$, and $f(a + h)$.
18. A spherical balloon with radius r inches has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of r inches to a radius of $r + 1$ inches.

- 19–22 ■ Evaluate the difference quotient for the given function. Simplify your answer.

19. $f(x) = 4 + 3x - x^2, \quad \frac{f(3+h) - f(3)}{h}$

20. $f(x) = x^3, \quad \frac{f(a+h) - f(a)}{h}$

21. $f(x) = \frac{1}{x}, \quad \frac{f(x) - f(a)}{x - a}$

22. $f(x) = \frac{x+3}{x+1}, \quad \frac{f(x) - f(1)}{x - 1}$

■ ■ ■ ■ ■ ■ ■ ■ ■ ■

- 23–27 ■ Find the domain of the function.

23. $f(x) = \frac{x}{3x - 1}$

24. $f(x) = \frac{5x + 4}{x^2 + 3x + 2}$

25. $f(t) = \sqrt{t} + \sqrt[3]{t}$

26. $g(u) = \sqrt{u} + \sqrt{4-u}$

27. $h(x) = \frac{1}{\sqrt[4]{x^2 - 5x}}$

■ ■ ■ ■ ■ ■ ■ ■ ■ ■

28. Find the domain and range and sketch the graph of the function $h(x) = \sqrt{4 - x^2}$.

- 29–40 ■ Find the domain and sketch the graph of the function.

29. $f(x) = 5$

30. $F(x) = \frac{1}{2}(x + 3)$

31. $f(t) = t^2 - 6t$

32. $H(t) = \frac{4 - t^2}{2 - t}$

33. $g(x) = \sqrt{x - 5}$

34. $F(x) = |2x + 1|$

35. $G(x) = \frac{3x + |x|}{x}$

36. $g(x) = \frac{|x|}{x^2}$

37. $f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ 1 - x & \text{if } x \geq 0 \end{cases}$

38. $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x \leq 2 \\ 2x - 5 & \text{if } x > 2 \end{cases}$

39. $f(x) = \begin{cases} x + 2 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$

40. $f(x) = \begin{cases} -1 & \text{if } x \leq -1 \\ 3x + 2 & \text{if } |x| < 1 \\ 7 - 2x & \text{if } x \geq 1 \end{cases}$

■ ■ ■ ■ ■ ■ ■ ■ ■ ■

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10 ■ CHAPTER 1 FUNCTIONS AND LIMITS

41–44 ■ Find an expression for the function whose graph is the given curve.

- 41. The line segment joining the points $(-2, 1)$ and $(4, -6)$
- 42. The line segment joining the points $(-3, -2)$ and $(6, 3)$
- 43. The bottom half of the parabola $x + (y - 1)^2 = 0$
- 44. The top half of the circle $(x - 1)^2 + y^2 = 1$

45–49 ■ Find a formula for the described function and state its domain.

- 45. A rectangle has perimeter 20 m. Express the area of the rectangle as a function of the length of one of its sides.
- 46. A rectangle has area 16 m^2 . Express the perimeter of the rectangle as a function of the length of one of its sides.
- 47. Express the area of an equilateral triangle as a function of the length of a side.
- 48. Express the surface area of a cube as a function of its volume.
- 49. An open rectangular box with volume 2 m^3 has a square base. Express the surface area of the box as a function of the length of a side of the base.

50. A taxi company charges two dollars for the first mile (or part of a mile) and 20 cents for each succeeding tenth of a mile (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in miles) for $0 < x < 2$, and sketch the graph of this function.

51. In a certain country, income tax is assessed as follows. There is no tax on income up to \$10,000. Any income over \$10,000 is taxed at a rate of 10%, up to an income of \$20,000. Any income over \$20,000 is taxed at 15%.

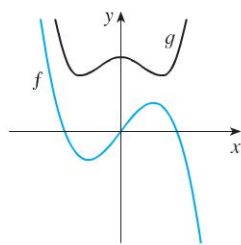
- (a) Sketch the graph of the tax rate R as a function of the income I .
- (b) How much tax is assessed on an income of \$14,000? On \$26,000?
- (c) Sketch the graph of the total assessed tax T as a function of the income I .

52. The functions in Example 6 and Exercises 50 and 51(a) are called *step functions* because their graphs look like

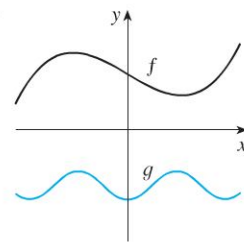
stairs. Give two other examples of step functions that arise in everyday life.

53–54 ■ Graphs of f and g are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

53.



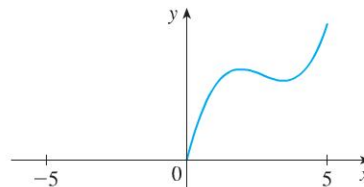
54.



- 55. (a) If the point $(5, 3)$ is on the graph of an even function, what other point must also be on the graph?
 (b) If the point $(5, 3)$ is on the graph of an odd function, what other point must also be on the graph?

56. A function f has domain $[-5, 5]$ and a portion of its graph is shown.

- (a) Complete the graph of f if it is known that f is even.
- (b) Complete the graph of f if it is known that f is odd.



57–62 ■ Determine whether f is even, odd, or neither. If you have a graphing calculator, use it to check your answer visually.

57. $f(x) = \frac{x}{x^2 + 1}$

58. $f(x) = \frac{x^2}{x^4 + 1}$

59. $f(x) = \frac{x}{x + 1}$

60. $f(x) = x|x|$

61. $f(x) = 1 + 3x^2 - x^4$

62. $f(x) = 1 + 3x^3 - x^5$

1.2 A CATALOG OF ESSENTIAL FUNCTIONS

In solving calculus problems you will find that it is helpful to be familiar with the graphs of some commonly occurring functions. These same basic functions are often used to model real-world phenomena, so we begin with a discussion of mathematical modeling. We also review briefly how to transform these functions by shifting, stretching, and reflecting their graphs as well as how to combine pairs of functions by the standard arithmetic operations and by composition.