Panel 1

<u>Antiderivatives</u>: The antiderivative F of a given function f is any function whose derivative is f, i.e. $F' = f$

<u>Ex</u>: $f(x) = x^2 \Rightarrow F(x) = \frac{1}{3}x^3$

i.e. $\frac{1}{3}x^3$ is an antideriv. of x^2

<u>Rule</u>: Antideriv of x^p is $\frac{1}{p+1}x^{p+1}$.

It is <u>best</u> to guess and check

Panel 2

<u>Ex</u>: Antideriv. of:

$f(x) = 3x^2 + 4x^3 + 2x^5$

$F(x) = x^3 + x^4 + 2 \cdot \frac{1}{6} x^6 \boxed{+ C}$

Antideriv. always should include a const.

$f(x) = 5\cos(x)$

$F(x) = 5\sin(x) + C$

$f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$

$F(x) = 2x^{1/2} + C$

Panel 3

Ex: Find the antiderivative of

$$y(x) = 4\sin(x) + \frac{2x^5 - \sqrt{x}}{x}$$

need anti- quotient rule!
Does not exist!

$$= 4\sin(x) + \frac{2x^5}{x} - \frac{\sqrt{x}}{x} =$$

$$= 4\sin(x) + 2x^4 - x^{-1/2}$$

$$G(x) = -4\cos(x) + 2\tfrac{1}{5}x^5 - 2x^{1/2} + C$$

check: $G' = 4\sin(x) + \frac{2}{\cancel{5}}\cancel{5}x^4 - \cancel{2}\frac{1}{\cancel{2}}x^{-1/2}$

Panel 4

More antiderivatives

$f(x) = \sec^2(x) \Rightarrow F(x) = \tan(x) + C$

$f(x) = \sec(x)\cdot\tan(x) \Rightarrow F(x) = \sec(x) + C$

$f(x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow F = \sin^{-1}(x) + C$

$f(x) = e^x \Rightarrow F = e^x + C$

$f(x) = \frac{\pi^2}{x} \Rightarrow F(x) = \underline{\pi^2 \cdot \ln|x|} + C$

x^{-1}

sin ✓	$\sin^{-1}$ ✓
cos ✓	$\cos^{-1}$ ✓
tan ✓	$\tan^{-1}$ ✓
sec ✓	
cot ✓	
csc ✓	
e^x ✓	
$\ln\|x\|$ ✓	

Panel 5

Find a function f s.t. $20 \cdot \frac{1}{1+x^2}$

$f'(x) = 4e^{2x} + 20(1+x^2)^{-1}$ and $f(0) = 2$

initial condition

$f(x) = 2e^{2x} + 20\tan^{-1}(x) + c$

Want $f(0) = 2$

$f(0) = 2e^{2\cdot 0} + 20\tan^{-1}(0) + c$

$= 2 + 0 + c = 2 \quad \Rightarrow c = 0$

$\tan^{-1}(0) = y$

$0 = \tan(y) \Rightarrow y = 0$

Panel 6

Ex. If $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ and $f(1) = 1$

Find $f(x)$.

Step 1: Find f' s.t. $f'' = 12x^2 + 6x - 4$

$f'(x) = 12\frac{1}{3}x^3 + 6\frac{1}{2}x^2 - 4x + c$

$= 4x^3 + 3x^2 - 4x + c$

$\Rightarrow f(x) = 4\frac{1}{4}x^4 + 3\frac{1}{3}x^3 - 4\frac{1}{2}x^2 + cx + d$

$= x^4 + x^3 - 2x^2 + cx + d$

$f' = 4x^3 + 3x^2 - 4x + c$ Know $f(0) = 4 = d$ $c = -3$

$f'' = 12x^2 + 6x - 4$ $f(1) = 1 = \cancel{1} + 1 - \cancel{2} + c + 4 = 1$

$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$ $c = -3$

Panel 7

A vehicle moves in a straight line and has acceleration $a(t)=6t+4$. Initial velocity is $v(0)=-6$ cm/sec and initial height is $s(0)=9$cm. Find $s(t)$.

$s(t)=t^3+2t^2-6t+9$

Know: s is distance

$\Rightarrow s'=v$ (velocity)

$v'=s''=a$ (acceleration)

$\Rightarrow a(t)=6t+4$ $\qquad s(t)=t^3+2t^2-6t+d$

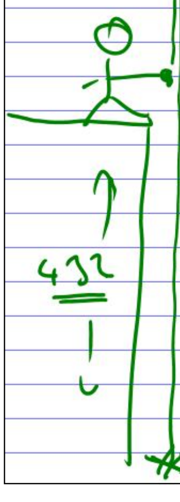
$\Rightarrow v(t)=3t^2+4t+\cancel{c}-6$ $\qquad s(0)=9=d$

Know: $v(0)=-6=c$

Panel 8

An object near the surface of the Earth is subject to a gravitational force that produces a downward acceleration denoted by g. For motion close to the ground we may assume that g is constant, its value being about 9.8 m/s^2 (or 32 ft/s^2).

EXAMPLE 6 A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?



$t=\frac{48}{32}=\frac{12}{8}=\frac{3}{2}$

After $t=\frac{3}{2}$ sec, ball is max height

$s(t)=0$:

$-16t^2+48t+432=0$

$\Rightarrow t=???$

$a(t)=-32$

$v(t)=-32t+c$, $v(0)=48=c$

$v(t)=-32t+48$

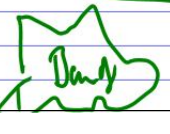
$s(t)=-16t^2+48t+d$, $s(0)=432$

$d=432$

$s(t)=-16t^2+48t+432$

$v(t)=-32t+48=0$

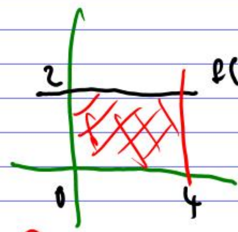
$48=32t$



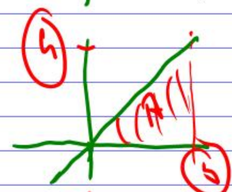
Panel 9

The Area Problem

Area under Curve:



$f(x)=2,\ x\in[0,4] \Rightarrow$ area is: 8

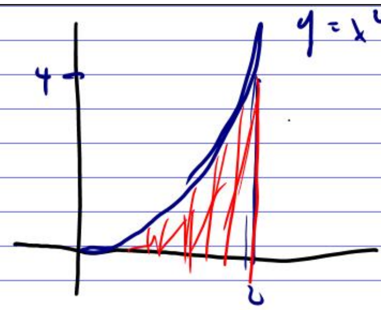


$A=\frac{1}{2}\cdot b\cdot h$

$A_1 = 4\cdot 1 = 4,\ A_2 = \frac{1}{2}$

$A = 4+\frac{1}{2} = \frac{9}{2}$

Panel 10

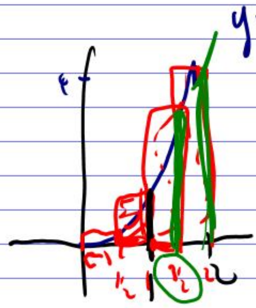


Could use graphing paper and count squares.
Create "box" with that shape, height 1cm. Fill with water, measure volume ⇒ area!

Different idea:

Panel 11

Area under Parabola



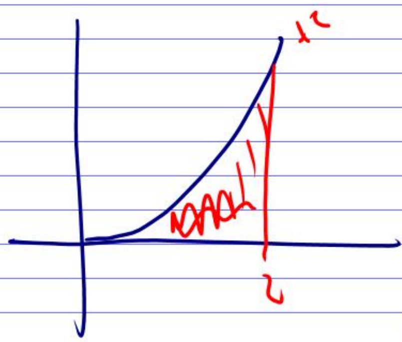
- Divide interval $[0,2]$ into 4 parts
- Make rectangles whose heights go to $f(x)=x^2$ on right side
- Area of these rectangles is

$$\frac{1}{2}\cdot f(\tfrac{1}{2})+\frac{1}{2} f(1)+\frac{1}{2} f(\tfrac{3}{2})+\frac{1}{2} f(2) =$$

$$=\frac{1}{2}\left(f(\tfrac{1}{2})+f(1)+f(\tfrac{3}{2})+f(2)\right) =$$

$$=\frac{1}{2}\left((\tfrac{1}{2})^2+(1)^2+(\tfrac{3}{2})^2+2^2\right) = \frac{1}{2}\cdot\left(\frac{1}{4}+1+\frac{9}{4}+4\right) = \frac{1}{2}\cdot\frac{15}{2} = \underline{\underline{\frac{15}{4}}}$$

Panel 12



Guess : $A = \frac{15}{4}$

true area $< \frac{15}{4}$

Better guess:

use 10 division

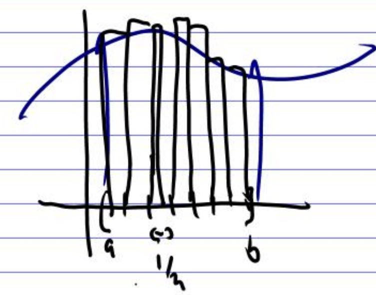
Better still, use 100 division, or 1000, or 10000

Panel 13

The Area under a function $f(x)$ is:

$\lim_{n\to\infty} \frac{1}{n}\left(f(x_1)+f(x_2)+\dots+f(x_n)\right)$

is area under f from a to b.



We write this as

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{1}{n}\left(f(x_1)+\dots+f(x_n)\right)$$

and call this definite integral

Panel 14

Definite Integral

$$\int_a^b f(x)\,dx$$ "integral from a to b of f(x)"

integrand

bounds

Indefinite Integrals

$$\int f(x)\,dx$$ "integral of f(x)"

Definitions $\int f(x)\,dx$ = Antiderivative

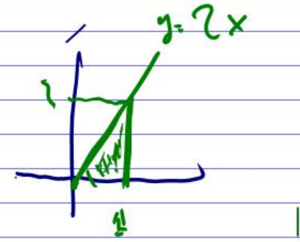
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \frac{1}{n}\left(f(x_1)+\dots+f(x_n)\right) = \text{area under } f(x)$$

Panel 15

Ex: Find

$$\int x^2 + e^x \, dx = \frac{1}{3}x^3 + e^x + c$$

$$\int_0^1 2x \, dx = \frac{1}{2} \cdot 1 \cdot 2 = 1$$



$$\int \frac{5}{x} \, dx = 5 \cdot \ln|x| + c$$

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

$$\int_{-2}^{2} \sqrt{4-x^2} \, dx = 2\pi$$