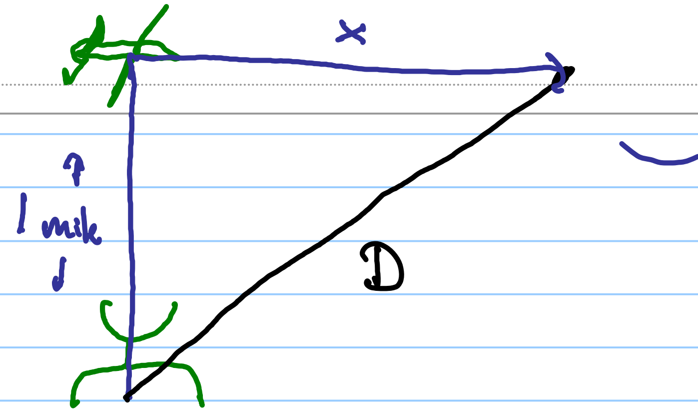
# Section 2.7




(4) A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

Know: $\frac{dx}{dt} = 500$

Want: $\frac{dD}{dt}$

Need: equation to relate D with x: $D^2 = x^2 + 1^2 = x^2 + 1$

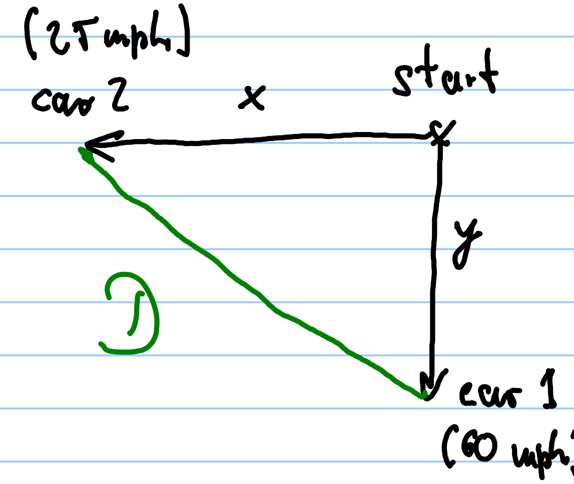
Diff both sides w. respect to t: $\cancel{2} D D' = \cancel{2} x x'$

$\Rightarrow D' = \frac{x \cdot x'}{D}$

Situation when $D = 2$ miles $\Rightarrow$ $2^2 = x^2 + 1$ $\Rightarrow$ $\underline{x = \sqrt{3}}$

Thus: $D' = \dfrac{x \cdot x'}{D} = \dfrac{\sqrt{3}\,500}{2} = \underline{250\sqrt{3}} \approx \underline{433}$

(13) Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?




$x$ = distance car 2
$y$ = distance car 1

Know: $x' = 60$, $y' = 25$ Want: $D'$

Need: equation that relates $D$, $x$, and $y$.

$\Rightarrow D^2 = x^2 + y^2$ $\qquad \Big| \frac{d}{dt}$

$\cancel{2}DD' = \cancel{2}xx' + 2yy'$ $\qquad \Rightarrow D' = \dfrac{xx' + yy'}{D}$

In 2 hours distance $x = 50$ m $(= 2 \cdot 25)$ and $y = 120$ $(= 2 \cdot 60)$.

$\Rightarrow D = \sqrt{50^2 + 120^2} = \sqrt{16900} = 130$

Thus: $D' = \dfrac{xx' + yy'}{D} = \dfrac{50 \cdot 25 + 120 \cdot 60}{130} = 65$ mph

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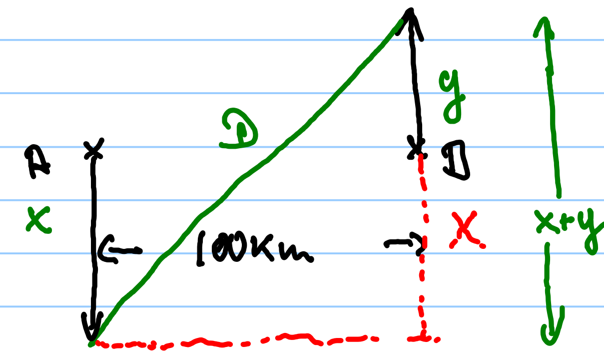
. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?




$x$ = distance ship A
$y$ = distance ship B

$D$ = distance between A and B

Know: $x' = 35$ mph, $y' = 25$ mph     Want: $D'$

Need: equation relating $D$, $x$, and $y$.

$$D^2 = 100^2 + (x+y)^2 = 100^2 + x^2 + 2xy + y^2 \quad \Big| \frac{d}{dt}$$

$\cancel{2}DD' = \cancel{2}xx' + \cancel{2}x'y + \cancel{2}xy' + \cancel{2}yy'$

$\Rightarrow D' = \frac{1}{D}(xx' + x'y + xy' + yy')$

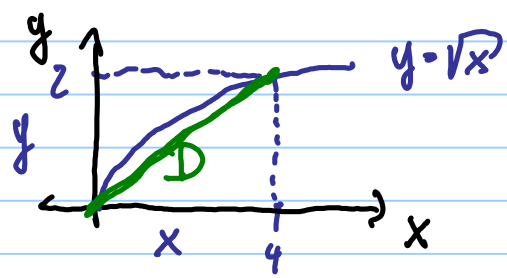
$\Rightarrow \underline{D'} = \frac{1}{260}(140 \cdot 35 + 35 \cdot 100 + 140 \cdot 25 + 100 \cdot 25) = \underline{\underline{55.38}}$

Ships start at noon
At 4 pm ship A
has traveled $4 \cdot 35 = 140 = x$
Ship B $4 \cdot 25 = 100 = y$

$\Rightarrow D^2 = 100^2 + (140 + 100)^2$

$\Rightarrow D = \sqrt{67600} = 260$

20 A particle is moving along the curve $y = \sqrt{x}$. As the particle passes through the point (4, 2), its $x$-coordinate increases at a rate of 3 cm/s. How fast is the distance from the particle to the origin changing at this instant?




<u>Know:</u> $x' = 3$ cm/sec <u>Want:</u> $D'$

<u>Need:</u> $D^2 = x^2 + y^2$ $\Rightarrow \cancel{2}DD' = \cancel{2}xx' + \cancel{2}yy'$. Need to eliminate $y$ and $y'$

But $y = \sqrt{x}$ so that $yy' = \sqrt{x} \cdot \frac{1}{2}\frac{1}{\sqrt{x}} \cdot x' = \frac{1}{2}x'$ $\Rightarrow$ $DD' = xx' + \frac{1}{2}x'$

Could have started with $D^2 = x^2 + y^2$ and subst $y = \sqrt{x}$:

$$D^2 = x^2 + (\sqrt{x})^2 = x^2 + x \Rightarrow 2DD' = 2xx' + x' \Rightarrow DD' = xx' + \frac{1}{2}x'.$$

If $x = 4$ and $y = 2$ we have $D = \sqrt{16+4} = \sqrt{20}$

$\Rightarrow$ $\underline{D'} = \frac{1}{\sqrt{20}}\left(4 \cdot 3 + 3/2\right) = \underline{\underline{3.019}}$

# Section 2.8

- Find the linearization $L(x)$ of the function at $a$.

1) $f(x) = x^4 + 3x^2, \quad a = -1$

2) $f(x) = 1/\sqrt{2+x}, \quad a = 0$

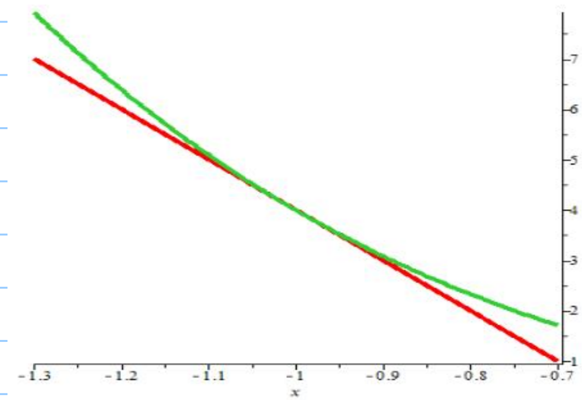
Linearization of $y = f(x)$ at $x = a$:

$$f(x) \approx f'(a)(x-a) + f(a)$$

1) $f(x) = x^4 + 3x^2 \qquad f'(x) = 4x^3 + 6x \qquad , \ a = -1$

$f(-1) = 4 \qquad f'(1) = -10$

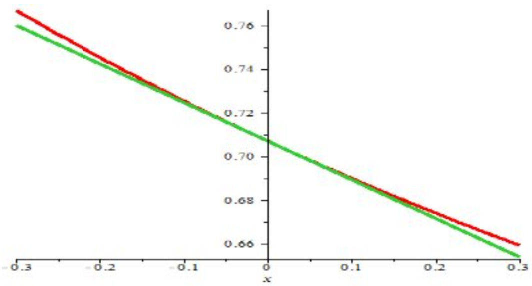
$$\Rightarrow f(x) \approx f'(-1)(x-a) + f(a) = -10(x+1) + 4$$



(2) $f(x) = \dfrac{1}{\sqrt{2+x}} = (2+x)^{-1/2}$ $\quad f'(x) = -\frac{1}{2}(2+x)^{-3/2}$, $a = 0$

$f(0) = 1/\sqrt{2}$ $\quad f'(0) = -\frac{1}{2} \cdot \frac{1}{2^{3/2}} = -\frac{1}{2^{5/2}}$

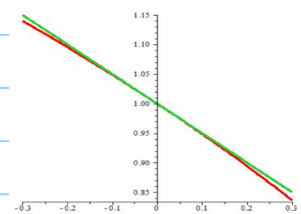
$\Rightarrow \underline{f(x)} \approx f'(0)(x-0) + f(0) = \underline{-1/2^{5/2}\, x + 1/2^{1/2}}$




(5) Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by graphing $f$ and the tangent line.

$f(x) = \sqrt{1-x}$, $f'(x) = -\frac{1}{2}(1-x)^{-1/2}$, $a = 0$

$\Rightarrow f(x) \approx -1/2\,(x-0) + 1 = \underline{-\frac{1}{2}x + 1}$



$\Rightarrow \underline{\sqrt{0.9}} = f(0.1) \approx -\frac{1}{2} \cdot 0.1 + 1 = 1 - 0.05 = \underline{0.95}$

$\sqrt{0.99} = f(0.01) \approx -\frac{1}{2}\, 0.01 + 1 = 1 - 0.005 = \underline{0.995}$

19 Let $y = \tan x$.
(a) Find the differential $dy$.
(b) Evaluate $dy$ and $\Delta y$ if $x = \pi/4$ and $dx = -0.1$.

Differential

$y = f(x) \Rightarrow dy = f'(x)\,dx$

$y = \tan(x) \Rightarrow dy = \sec^2(x)\,dx$. If $x = \pi/4$ and $dx = -0.1$ then

$dy = \sec^2(\pi/4)(-0.1) =$

20. Let $y = \sqrt{x}$.
(a) Find the differential $dy$.
(b) Evaluate $dy$ and $\Delta y$ if $x = 1$ and $dx = \Delta x = 1$.
(c) Sketch a diagram like Figure 5 showing the line segments with lengths $dx$, $dy$, and $\Delta y$.

$$dy = \frac{1}{2} x^{-1/2} dx$$

$$dy = \frac{1}{2} (1)^{-1/2} \cdot 1 = \underline{\underline{1/2}}$$

21. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing (a) the volume of the cube and (b) the surface area of the cube.

$$V = x^3 \Rightarrow dV = 3x^2 dx$$

$$V = 30^3 = 2700 \qquad = 3 \cdot 30^2 \cdot 0.1 = 270$$

$\Rightarrow$ relative error in $x$: $\frac{0.1}{30} = 0.003 = \underline{0.3\%}$

in $V$: $\frac{270}{27000} = 0.01 = \underline{1\%}$

$$S = 6 \cdot x^2 \Rightarrow dS = 12x\, dx$$

$$\Rightarrow dS = 12 \cdot 30 \cdot 0.1 = 36$$

$\Rightarrow$ relative error in $S$ is $\frac{dS}{S} = \frac{36}{6 \cdot 30^2} = 0.0067 = \underline{0.7\%}$

22 The radius of a circular disk is given as 24 cm with a maximum error in measurement of 0.2 cm.
(a) Use differentials to estimate the maximum error in the calculated area of the disk.
(b) What is the relative error? What is the percentage error?

23 The circumference of a sphere was measured to be 84 cm with a possible error of 0.5 cm.
(a) Use differentials to estimate the maximum error in the calculated surface area. What is the relative error?
(b) Use differentials to estimate the maximum error in the calculated volume. What is the relative error?

$A = \pi r^2 = \pi\, 24^2 = 1809.57$

$dA = \pi\, 2r \cdot dr = \pi \cdot 2 \cdot 24 \cdot 0.2 = 30.16$

Relative error in $r$: $\frac{0.2}{24} = 0.008 = 0.8\%$

in $A$: $\frac{dA}{A} = \frac{30.16}{1809.57} = 0.016 = 1.6\%$

$r = 42$

Surface of sphere, $S = 4\pi r^2$, $dS = 8\pi \underbrace{r}_{42}\, \underbrace{dr}_{=0.5} = 527.8$; rel. error: $\frac{dS}{S} = \frac{527.8}{22167} = 0.02 = 2\%$

Volume of sphere, $V = \frac{4}{3}\pi r^3$, $dV = 4\pi \underbrace{r^2}_{42^2}\, \underbrace{dr}_{0.5} = 11083$; rel. error $\frac{dV}{V} = \frac{11083}{310339} = 0.035 = 3.5\%$