Panel 1

Math 1401: Last time

Details: grading, web site, DyKnow

http://pirate.shu.edu/~wachsmut/

Functions, domain, range

Graphs, vertical line test

Piecewise defined function

Function review:

even, odd

Panel 2

Dyknow Interactive ①

What is Calculus:

a) boring course to take because it is required

b) difficult course to pass by cheating

c) exciting and challenging course to expand my mathematical horizont and to stimulate my mathematical senses

d) calculus - never heard of it

Panel 3

Dyknow Interactive ②

Solve $8x - x(x+3) = 4x(1-x)$

① Expand: $8x - x^2 - 3x = 4x - 4x^2$

② Collect:

$x + 3x^2 = 0$

③ Solve:

$x(1 + 3x) = 0$

$x = 0$

$x = -1/3$

Panel 4

$f(x) = 4 + 3x - x^2$

$\frac{f(3+h) - f(3)}{h}$

$f(3) = 4 + 3 \cdot 3 - 3 \cdot 3 = 4$

$f(3+h) = 4 + 3(3+h) - (3+h)^2 =$

$= 4 + 9 + 3h - 9 - 6h - h^2$

$= 4 - 3h - h^2$

$\frac{f(3+h) - f(3)}{h} = \frac{\cancel{4} - 3h - h^2 - \cancel{4}}{h} = \frac{h(-3-h)}{\cancel{h}} = -3 - h$

$f(x) = \frac{x}{3x-1}$ Domain: No good: $3x - 1 = 0$

$\{x \neq 1/3\}$ $\Leftarrow$ $\Leftrightarrow x = 1/3$

Panel 5

even

odd

TIONS AND LIMITS

or the function whose graph is the

the points $(-2, 1)$ and $(4, -6)$

the points $(-3, -2)$ and $(6, 3)$

$$f(x) = \frac{x}{x^2+1} \qquad f(-x) = \frac{-x}{x^2+1} = -\frac{x}{x^2+1} = -f(x) \quad \text{odd}$$

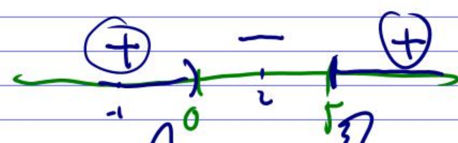
Panel 6

$$f = \frac{1}{\sqrt[4]{x^2 - 5x}}$$

Need: $x^2 - 5x > 0$

$x^2 - 5x = 0$

$x(x-5) = 0, \quad x = 0, 5$



$(-\infty, 0) \cup (5, \infty)$

Panel 7

Name:

Quiz #1:

① What is the domain of the function $f(x) = \dfrac{3}{x^2 - 2x}$

② If $f(x) = 2x^2 + 1$, find

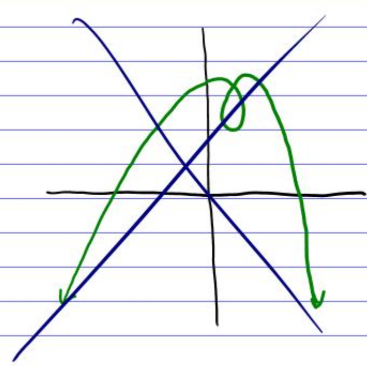
a) $f(-1)$

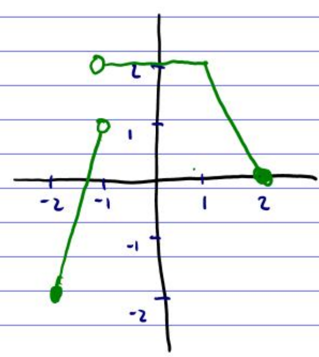
b) $f(2s)$

Panel 8

Quiz #1 - part 2 -

③ Consider the graphs below. Cross out the one that is not a function. For the other, list domain and range.





Domain: $[-2, 2] - \{-1\}$

Range: $[-2, 2)$

Panel 9

We now have a basic understanding of basic functions.

limits!!!

Panel 10

Most important concept in Calculus: limit

$f(x) = \frac{1}{x}$ all $x \neq 0$

What if $x \neq 0$ but very close to 0?

x	$f(x)$
0.1	$1/_{1/10} = 10$
0.01	$1/_{1/100} = 100$
0.001	1000
-0.1	-10
-0.01	-100
	-1000

as x gets closer to zero, f(x) gets bigger

Panel 11

$f(x) = \frac{\sin(x)}{x}$, domain $x \neq 0$

x	$f(x)$
0.1	0.998
0.01	0.99998
0.001	0.9999998
0.0001	⋮
	1

as x gets closer to zero,
$f(x)$ gets closer to 1.
But $f(0)$ is still undefined

Panel 12

Def: "The limit of $f(x)$ as x approaches a is L"
means: as x gets closer to a, $f(x)$ gets closer to L.
We write

$$\lim_{x \to a} f(x) = L$$

Note:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$\lim_{x \to 0} \frac{1}{x}$ is undefined

$$\lim_{x \to 2} x^2 - x + 2 = 4$$

x	
1.9	
1.99	
1.999	
⋮	4

Panel 13

Ex: $\lim_{x \to 1} \frac{x-1}{x^2-1} = 1/2$ pattern? for loosers!

$\lim_{x \to 1} \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}} = \lim_{x \to 1} \frac{1}{x+1} = 1/2$ ✓

$= 1/6$

Ex: $\frac{1}{6} = \lim_{x \to 0} \frac{(\overset{a}{\sqrt{x^2+9}} - \overset{b}{3})(\overset{a}{\sqrt{x^2+9}} + \overset{b}{3})}{x^2 \, (\sqrt{x^2+9}+3)} = \frac{(\cancel{x^2} + \cancel{9} - \cancel{9})}{\cancel{x^2}(\sqrt{x^2+9}+3)} = \lim_{x \to 0} \frac{1}{\sqrt{x^2+9}+3}$

$(a+b)(a-b) = a^2 - b^2$

$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$ pattern! $\sin\left(\frac{\pi}{\ }\right) =$

$x = 0.0000001 = \frac{1}{10000000}$

$= \sin(10000000\pi) = 0$

Panel 14

Why do we have to look for a pattern? Just use one really "close" number and get it over with?

$= \lim_{x \to 0} \sin\left(\frac{\pi}{x}\right)$, $x = \frac{1}{1000000} \Rightarrow \sin\left(\frac{\pi}{x}\right) = \sin(1000000\pi) = 0$

$x = \frac{2}{1000000} \Rightarrow \sin\left(\frac{\pi}{x}\right) = \sin\left(\frac{1000000\,\pi}{2}\right)$

$= \pm 1$

no pattern,
no limit!

x	$f(x)$
	0
	1
	-1
	0
	-1
	-1

Panel 15

Limits come in 3 Varieties:

$\frac{0}{\#} = 0$

$\frac{\#}{0}$ = undef.

$\frac{0}{0}$ = more work

tricky

5 4

$\lim_{x \to 1} \frac{x^2-1}{x^2-2} = \frac{0}{-1} = 0$

$\lim_{x \to 1} \frac{x^2-2}{x-1} = \frac{-1}{0}$ d.n.e

$\lim_{x \to 1} \frac{x^2-1}{x-1} = \frac{0}{0} =$

$\lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$

Panel 16

Even More Examples

more work

x^3-3x+2

$\lim_{x \to 1} \frac{x^3-3x+2}{x^2-1} = \frac{0}{0} = \lim_{x \to 1} \frac{(x-1)^{\cancel{2}}(x+2)}{\cancel{(x-1)}(x+1)} = \lim_{x \to 1} \frac{(x-1)(x+2)}{(x+1)} = 0$

more work

$\lim_{x \to 0} \frac{x^3+x^2-6x}{x^2} = \frac{0}{0} = \lim_{x \to 0} \frac{x(x^2+x-6)}{x^2} = \lim_{x \to 0} \frac{\cancel{x}(x+3)(x-2)}{x^{\cancel{2}}}$

$= -\frac{6}{0}$ undef.