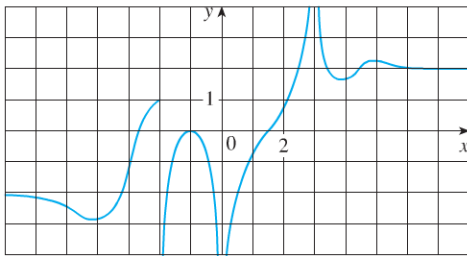


2. For the function  $g$  whose graph is given, state the following.
- (a)  $\lim_{x \rightarrow \infty} g(x)$       (b)  $\lim_{x \rightarrow -\infty} g(x)$   
 (c)  $\lim_{x \rightarrow 3} g(x)$       (d)  $\lim_{x \rightarrow 0} g(x)$   
 (e)  $\lim_{x \rightarrow -2^+} g(x)$       (f) The equations of the asymptotes



**3–8** ■ Sketch the graph of an example of a function  $f$  that satisfies all of the given conditions.

3.  $f(0) = 0$ ,  $f(1) = 1$ ,  $\lim_{x \rightarrow \infty} f(x) = 0$ ,  $f$  is odd
4.  $\lim_{x \rightarrow 0^+} f(x) = \infty$ ,  $\lim_{x \rightarrow 0^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = 1$ ,  
 $\lim_{x \rightarrow -\infty} f(x) = 1$
5.  $\lim_{x \rightarrow 2} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = \infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 0$ ,  
 $\lim_{x \rightarrow 0^+} f(x) = \infty$ ,  $\lim_{x \rightarrow 0^-} f(x) = -\infty$
6.  $\lim_{x \rightarrow -2} f(x) = \infty$ ,  $\lim_{x \rightarrow -\infty} f(x) = 3$ ,  $\lim_{x \rightarrow \infty} f(x) = -3$
7.  $f(0) = 3$ ,  $\lim_{x \rightarrow 0^-} f(x) = 4$ ,  $\lim_{x \rightarrow 0^+} f(x) = 2$ ,  
 $\lim_{x \rightarrow -\infty} f(x) = -\infty$ ,  $\lim_{x \rightarrow 4^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow 4^+} f(x) = \infty$ ,  
 $\lim_{x \rightarrow \infty} f(x) = 3$
8.  $\lim_{x \rightarrow 3} f(x) = -\infty$ ,  $\lim_{x \rightarrow \infty} f(x) = 2$ ,  $f(0) = 0$ ,  $f$  is even

9. Guess the value of the limit

$$\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$$

by evaluating the function  $f(x) = x^2/2^x$  for  $x = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 50$ , and  $100$ . Then use a graph of  $f$  to support your guess.

10. Determine  $\lim_{x \rightarrow 1^-} \frac{1}{x^3 - 1}$  and  $\lim_{x \rightarrow 1^+} \frac{1}{x^3 - 1}$
- (a) by evaluating  $f(x) = 1/(x^3 - 1)$  for values of  $x$  that approach 1 from the left and from the right,  
 (b) by reasoning as in Example 1, and  
 (c) from a graph of  $f$ .



11. Use a graph to estimate all the vertical and horizontal asymptotes of the curve

$$y = \frac{x^3}{x^3 - 2x + 1}$$

12. (a) Use a graph of

$$f(x) = \left(1 - \frac{2}{x}\right)^x$$

to estimate the value of  $\lim_{x \rightarrow \infty} f(x)$  correct to two decimal places.

(b) Use a table of values of  $f(x)$  to estimate the limit to four decimal places.

**13–31** ■ Find the limit.

13.  $\lim_{x \rightarrow -3^+} \frac{x+2}{x+3}$       14.  $\lim_{x \rightarrow 5^-} \frac{6}{x-5}$
15.  $\lim_{x \rightarrow 1} \frac{2-x}{(x-1)^2}$       16.  $\lim_{x \rightarrow \pi^-} \cot x$
17.  $\lim_{x \rightarrow (-\pi/2)^-} \sec x$       18.  $\lim_{x \rightarrow \infty} \frac{3x+5}{x-4}$
19.  $\lim_{x \rightarrow \infty} \frac{x^3+5x}{2x^3-x^2+4}$       20.  $\lim_{t \rightarrow -\infty} \frac{t^2+2}{t^3+t^2-1}$
21.  $\lim_{u \rightarrow \infty} \frac{4u^4+5}{(u^2-2)(2u^2-1)}$       22.  $\lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{9x^2+1}}$
23.  $\lim_{x \rightarrow \infty} (\sqrt{9x^2+x} - 3x)$
24.  $\lim_{x \rightarrow \infty} (\sqrt{x^2+ax} - \sqrt{x^2+bx})$
25.  $\lim_{x \rightarrow \infty} \cos x$       26.  $\lim_{x \rightarrow \infty} \frac{\sin^2 x}{x^2}$
27.  $\lim_{x \rightarrow \infty} (x - \sqrt{x})$       28.  $\lim_{x \rightarrow \infty} \frac{x^3 - 2x + 3}{5 - 2x^2}$
29.  $\lim_{x \rightarrow -\infty} (x^4 + x^5)$       30.  $\lim_{x \rightarrow \infty} (x^2 - x^4)$
31.  $\lim_{x \rightarrow \infty} \frac{x + x^3 + x^5}{1 - x^2 + x^4}$

32. (a) Graph the function

$$f(x) = \frac{\sqrt{2x^2+1}}{3x-5}$$

How many horizontal and vertical asymptotes do you observe? Use the graph to estimate the values of the limits

$$\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2+1}}{3x-5} \quad \text{and} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2+1}}{3x-5}$$

(b) By calculating values of  $f(x)$ , give numerical estimates of the limits in part (a).