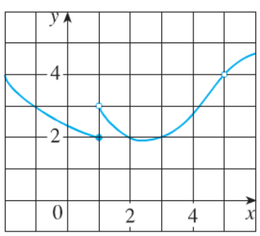
1.3 EXERCISES

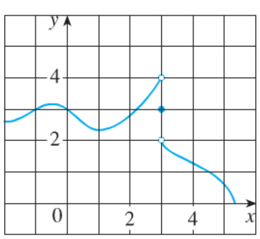
1. If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$.
(a) Find the average velocity for the time period beginning when $t = 2$ and lasting
(i) 0.5 second (ii) 0.1 second
(iii) 0.05 second (iv) 0.01 second
(b) Estimate the instantaneous velocity when $t = 2$.

2. If an arrow is shot upward on the moon with a velocity of 58 m/s, its height in meters t seconds later is given by $h = 58t - 0.83t^2$.
(a) Find the average velocity over the given time intervals:
(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$
(iv) $[1, 1.01]$ (v) $[1, 1.001]$
(b) Estimate the instantaneous velocity when $t = 1$.

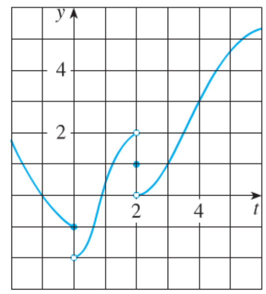
3. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.
(a) $\lim_{x\to 1^-} f(x)$ (b) $\lim_{x\to 1^+} f(x)$ (c) $\lim_{x\to 1} f(x)$
(d) $\lim_{x\to 5} f(x)$ (e) $f(5)$



4. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.
(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$
(d) $\lim_{x\to 3} f(x)$ (e) $f(3)$



5. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.
(a) $\lim_{t\to 0^-} g(t)$ (b) $\lim_{t\to 0^+} g(t)$ (c) $\lim_{t\to 0} g(t)$
(d) $\lim_{t\to 2^-} g(t)$ (e) $\lim_{t\to 2^+} g(t)$ (f) $\lim_{t\to 2} g(t)$
(g) $g(2)$ (h) $\lim_{t\to 4} g(t)$



6. Sketch the graph of the following function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists:

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ (x-1)^2 & \text{if } x \geqslant 1 \end{cases}$$

7–10 ▪ Sketch the graph of an example of a function f that satisfies all of the given conditions.

7. $\lim_{x\to 1^-} f(x) = 2$, $\lim_{x\to 1^+} f(x) = -2$, $f(1) = 2$

8. $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = -1$, $\lim_{x\to 2^-} f(x) = 0$,
$\lim_{x\to 2^+} f(x) = 1$, $f(2) = 1$, $f(0)$ is undefined

9. $\lim_{x\to 3^+} f(x) = 4$, $\lim_{x\to 3^-} f(x) = 2$, $\lim_{x\to -2} f(x) = 2$,
$f(3) = 3$, $f(-2) = 1$

10. $\lim_{x\to 1} f(x) = 3$, $\lim_{x\to 4^-} f(x) = 3$, $\lim_{x\to 4^+} f(x) = -3$,
$f(1) = 1$, $f(4) = -1$

11–14 ▪ Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

11. $\lim_{x\to 2} \dfrac{x^2 - 2x}{x^2 - x - 2}$
$x = 2.5, 2.1, 2.05, 2.01, 2.005, 2.001,$
$1.9, 1.95, 1.99, 1.995, 1.999$

12. $\lim_{x\to -1} \dfrac{x^2 - 2x}{x^2 - x - 2}$
$x = 0, -0.5, -0.9, -0.95, -0.99, -0.999,$
$-2, -1.5, -1.1, -1.01, -1.001$

13. $\lim_{x\to 0} \dfrac{\sin x}{x + \tan x}$
$x = \pm 1, \pm 0.5, \pm 0.2, \pm 0.1, \pm 0.05, \pm 0.01$