Panel 1

Integration Techniques

Diff — always works

Int — guessing; sometimes does not work

power rule — anti-power rule ✓

chain — anti-chain (Substitution) ×

product — anti-product (Integration by Parts)

quotient — X

1

Panel 2

Integration Techniques:

Differentiation — Integration

2

Panel 3

Integration by Substitution

$$\int f(u(x))\,u'(x)\,dx = \int f(u)\,du$$

Ex: $\int 3\sin(3x)\,dx$

$u = 3x$ | $u = \sin(3x)$

$\frac{du}{dx} = 3$ | $\frac{du}{dx} = 3\cos(3x)$

$du = 3dx$ | $du = 3\cos(3x)\,dx \rightarrow 3dx = \frac{du}{\cos(3x)}$

$\int 3\sin(u)\,dx = \int \sin(u)\,du$ | $\int 3u\,dx = \int \cancel{\frac{1}{\cos(3x)}}\, u\,du$

$-\cos(u) = -\cos(3x) + c$ | need only u's!

3

Panel 4

Substitution:

① Define $u = \ldots$ — usually stuff in parenthesis

② Compute $\frac{du}{dx}$ and solve for du

③ Rewrite integral: only u's left

④ Try to integrate now → re-substitute when done

Ex: $\int (2x+1)\cos(x^2+x)\,dx = \int \cos(u)\,du = \sin(u) + C$

$= \sin(x^2+x) + C$

$u = 2x+1$ $\quad du = 2\,dx$ ✗

$u = x^2+x$ $\quad du = (2x+1)\,dx$ ✓

$u = \cos(x^2+x)$ $\quad du = -\sin(x^2+x)\cdot(2x+1)\,dx$ ✗

4

Panel 5

Find the following indefinite integrals:

$$\int \cos(x)\cdot \sin^3(x)\,dx \sim \sin(x)\cdot\sin^2(x)\,dx$$

$u=\cos(x) \Rightarrow du = -\sin(x)\,dx$; $-\int u \sin^2(x)\,du = -\int u\,(1-\cos^2(x))\,du = -\int u(1-u^2)\,du$

$\cancel{u=}\sin^3(x) \Rightarrow du = 3\sin^2(x)\cdot\cos(x)\,dx$

$\cancel{u=}x \Rightarrow du = 1\cdot dx$: $\int \cos(u)\sin^3(u)\,du$ X

$u=\sin(x) \Rightarrow du = \cos(x)\,dx$: $\int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}\sin^4(x) + C$

$$\int \frac{2x+1}{x^2+x+8}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x^2+x+8| + C$$

X $u = 2x+1$

$u = x^2+x+8 \Rightarrow du = (2x+1)\,dx$

$\cancel{u} = \frac{2x+1}{x^2+x+8}$

5

Panel 6

$$\int (5x+7)^{10}\,dx = \frac{1}{5}\int u^{10}\,du = \frac{1}{5}\frac{1}{11}u^{11} + C = \frac{1}{55}(5x+7)^{11} + C$$

$u = 5x+7$

$du = 5\,dx \Leftrightarrow \frac{1}{5}du = dx$

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|\cos(x)| + C$$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

X $u = \sin(x) \Rightarrow du = \cos(x)\,dx$

✓ $u = \cos(x) \Rightarrow du = -\sin(x)\,dx$

6

Panel 7

Substitution Method for definite integrals:

bounds $\int_0^2 3x^2 \sin(x^3)\,dx$

2 choices: 1. Substitute, but keep the bounds
Then re-substitute and use original bounds

$$\int_0^2 3x^2 \sin(x^3)\,dx = \int_{x=0}^{x=2} \sin(u)\,du = -\cos(u)\Big|_{x=0}^{x=2} =$$

$u = x^3$

$du = 3x^2 dx$

$$-\cos(x^3)\Big|_0^2 = -\cos(8) + \cos(0) = 1 - \cos(8)$$

7

Panel 8

$\int_0^2 3x^2 \sin(x^3)\,dx$ preferred

2 choice: Substitute as usual. Also substitute bounds. Then plug in without resubstitution

$$\int_0^2 3x^2 \sin(x^3)\,dx = \int_0^8 \sin(u)\,du = -\cos(u)\Big|_0^8 = -\cos(8) + \cos(0) = 1 - \cos(8)$$

$u = x^3$

$du = 3x^2\,dx$ 8

if $x = 0 \Rightarrow u = 0$

if $x = 2 \Rightarrow u = 8$

8

Panel 9

Ex: $\int_1^2 \frac{dx}{(3-5x)^2} = -\frac{1}{5}\int \frac{1}{u^2}\,du \longrightarrow -\frac{1}{5}\int_{x=1}^{x=2} \frac{1}{u^2}\,du =$

$+\frac{1}{5}u^{-1}\Big|_{x=1}^{x=2} =$

$u = 3-5x$

$du = -5\,dx \Rightarrow -\frac{1}{5}du = dx$

if $x=1 \Rightarrow u = 3-5 = -2$

if $x=2 \Rightarrow u = 3-10 = -7$

$\frac{1}{5}(3-5x)^{-1}\Big|_1^2 =$

$\frac{1}{5}(3-10)^{-1} - \frac{1}{5}(3-5)^{-1}$

$-\frac{1}{5}\int_{-2}^{-7} \frac{1}{u^2}\,du = \frac{1}{5}u^{-1}\Big|_{-2}^{-7} = \frac{1}{5}(-7)^{-1} - \frac{1}{5}(-2)^{-1}$

Panel 10

Sometimes one method is preferred:

$\int_{-1}^{1} 3xe^{x^2}\,dx = \frac{3}{2}\int_{x=-1}^{x=1} e^u\,du = \frac{3}{2}e^u\Big|_{x=-1}^{x=1} = \frac{3}{2}e^{x^2}\Big|_{-1}^{1}$

$= \frac{3}{2}e^1 - \frac{3}{2}e^1 = 0$

$u = x^2$

$du = 2\boxed{x\,dx}$

$\Rightarrow x\,dx = \frac{1}{2}du$

$\int_{-1}^{1} 3xe^{x^2}\,dx = \int_1^1 \_\_\_ = 0$

$u = x^2$

$du = 2x\,dx$

if $x=-1 \Rightarrow u=1$

if $x=1 \Rightarrow u=1$

Panel 11

Tricky Substitutions

$\int x(x+1)\,dx$ ✓

$\int x\sqrt{x+1}\,dx$ $\boxed{\text{Inter}}$ $x=u+1$

no simplify

stuck

$\int \frac{x-1}{x+1}\,dx$

$u=x-1$
$du=dx$

$\int \frac{u}{x+1}\,du = \int \frac{u}{u+2}\,du$

$u=x+1$
$du=1dx$
$x=u-1$

$\int \frac{x-1}{u}\,du = \int \frac{u-2}{u}\,du$

$= \int \cancel{1}\frac{u}{u} - \frac{2}{u}\,du = u-2\ln|u|$