Panel 1

Last Time

ℓ'

1

Panel 2

Name: _____

Quiz 12

① A cell culture, prepared at noon with 100 cells, grew to 400 cells 2 hours later. How many cells will the culture contain at 6 pm, assuming exponential growth?

2

Panel 3

② Find the derivatives of the following functions.

a) $f(x) = \ln(x) \cdot e^{3x^2}$

b) $g(x) = \sin^{-1}(1-x) + \cos^{-1}(1-x) = \pi/2$

$\sin^{-1}(x) + \cos^{-1}(x) = \text{const}$

$\sin^{-1}(0) + \cos^{-1}(0) = \pi/2$

0

$g'(x) = \frac{-1}{\sqrt{1-(1-x)^2}} + \frac{+1}{\sqrt{1-(1-x)^2}} = 0$

$\sin^{-1}(0) = y$

$0 = \sin(y)$

$\cos^{-1}(0) = y$

$0 = \cos(y)$

c) $h(x) = \tan^{-1}\left(e^{x^2}\right)$

$\frac{1}{1+(e^{x^2})^2} e^{x^2} \cdot 2x$

Panel 4

Application of Antiderivative (Physics)

A vehicle moves in a straight line and has acceleration $a(t) = 6t + 4$. Initial velocity is $v(0) = -6$ cm/sec and initial height is $s(0) = 9$ cm. Find $s(t)$.

$s(t) = \text{dist.}$

$\Rightarrow s'(t) = v(t)$

$= s''(t) = v'(t) = a(t)$

$v(t) = \int 6t+4\,dt = 6\frac{1}{2}t^2 + 4t + C$

$= 3t^2 + 4t + C$

$v(0) = -6 = 3 \cdot 0^2 + 4 \cdot 0 + C \Rightarrow C = -6$

$s(t) = \int 3t^2 + 4t - 6\,dt = s(t) = t^3 + 2t^2 - 6t + 9$

$3 \cdot \frac{1}{3}t^3 + 4\frac{1}{2}t^2 - 6t + d$, $s(0) = 9$

Panel 5

An object near the surface of the Earth is subject to a gravitational force that produces a downward acceleration denoted by g. For motion close to the ground we may assume that g is constant, its value being about 9.8 m/s^2 (or 32 ft/s^2).

EXAMPLE 6 A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

$a(t) = -g$

$v(t) = \int -g\,dt = -gt + C = -gt + 48$

$v(0) = -g\cdot 0 + C = C = 48$

$s(t) = \int -gt + 48\,dt = -g\frac{1}{2}t^2 + 48t + C$

$s(0) = 432$

$\Rightarrow s(t) = -\frac{g}{2}t^2 + 48t + 432$

hit ground: $s(t) = 0$

max height: $v(t) = 0$

$\Rightarrow s\left(\frac{48}{g}\right)$

5

Panel 6

$a = -32$

$v(t) = -32t + C = -32t + 0$

$s(t) = -16t^2 + d$

say it takes 3 secs to hit the ground!

$s(3) = 0 = -16\cdot 9 + d$

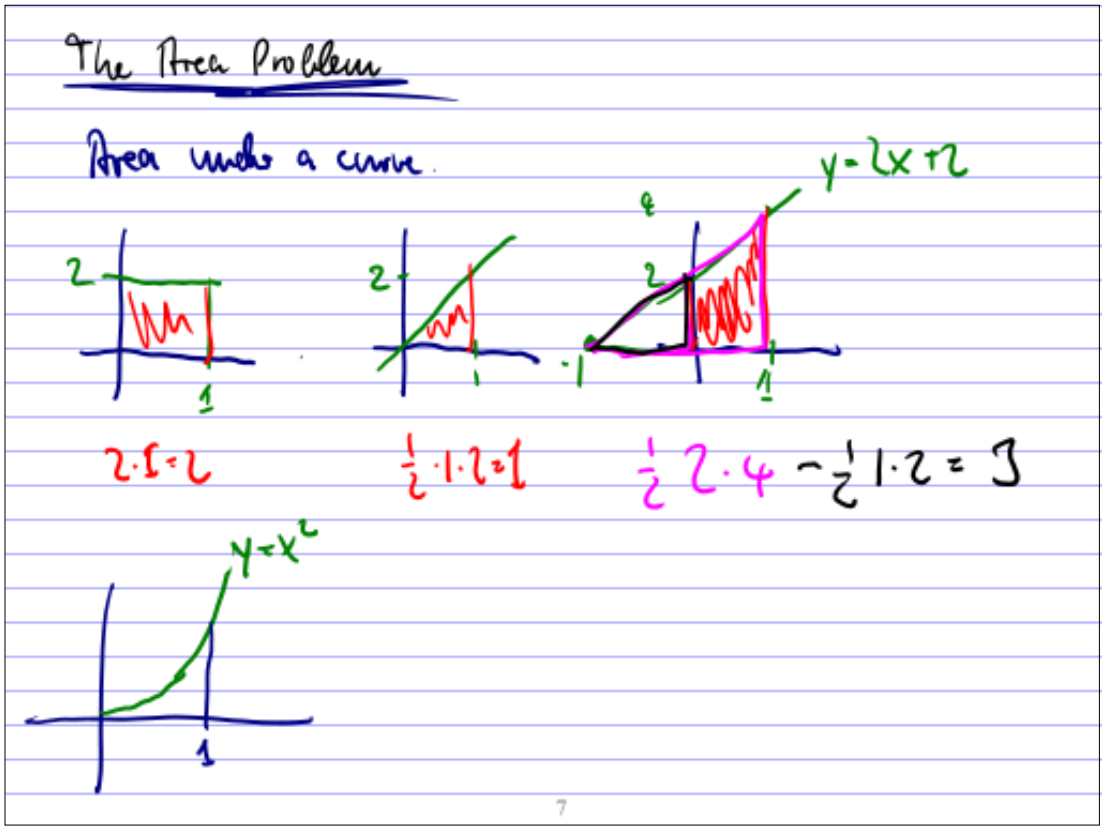
$d = 16\cdot 9$

The height is $3^2\cdot 16$

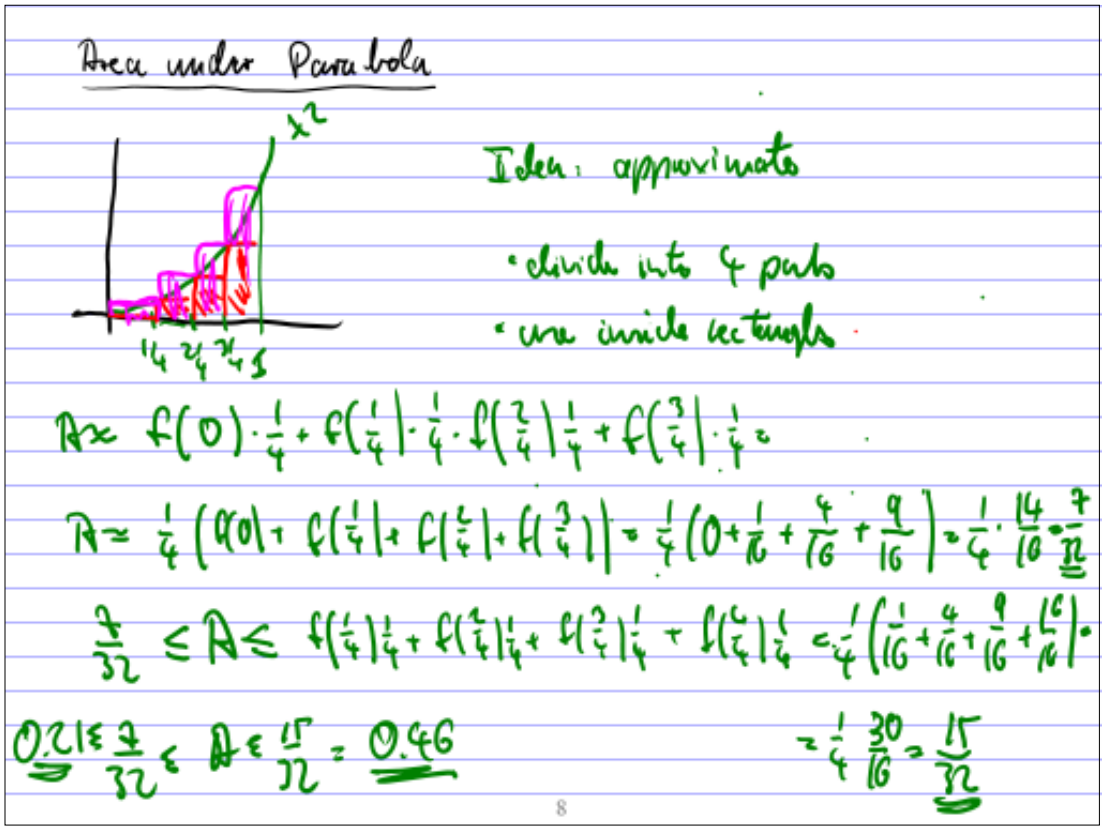
Down after 3 secs

6

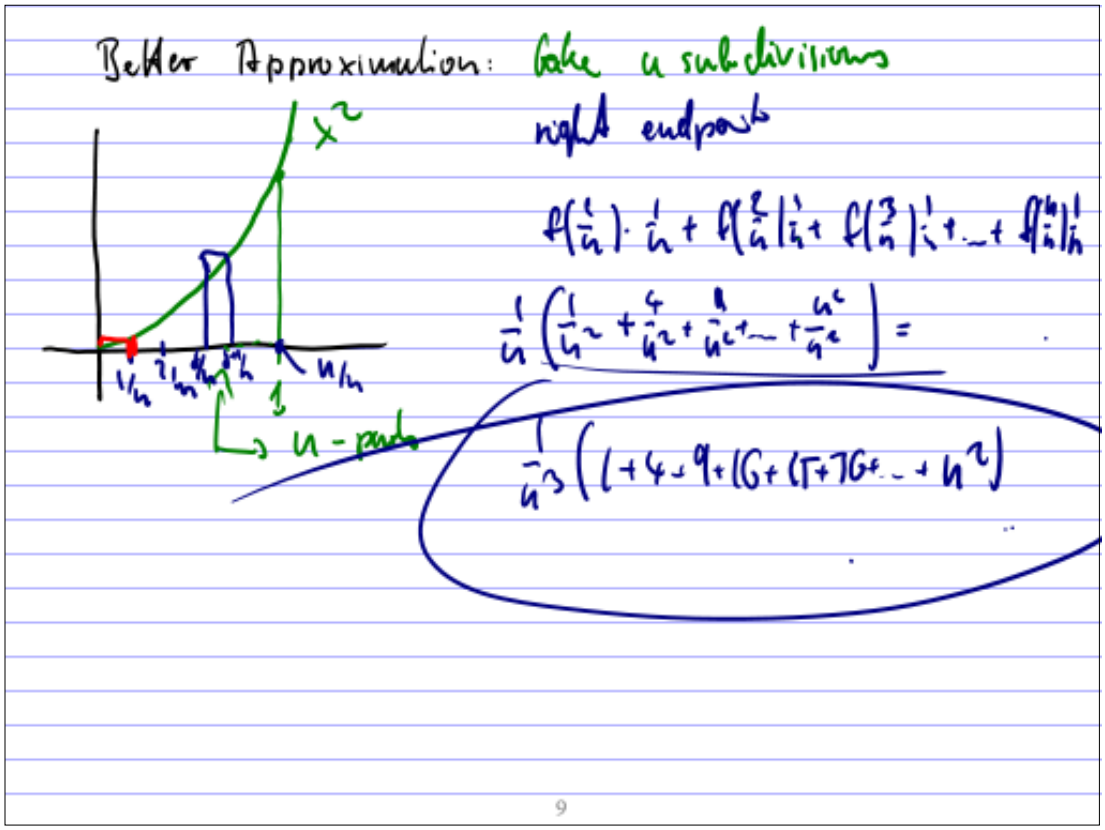
Panel 7



Panel 8



Panel 9


Better Approximation: take n subdivisions
right endpoints
x^2
$\frac{1}{n}$ $\frac{2}{n}$ $\frac{3}{n}$ $\frac{n}{n}$
n - parts
$f(\frac{1}{n})\cdot\frac{1}{n} + f(\frac{2}{n})\frac{1}{n} + f(\frac{3}{n})\frac{1}{n} + \dots + f(\frac{n}{n})\frac{1}{n}$
$\frac{1}{n}\left(\frac{1}{n^2} + \frac{4}{n^2} + \frac{9}{n^2} + \dots + \frac{n^2}{n^2}\right) =$
$\frac{1}{n^3}\left(1 + 4 + 9 + 16 + 25 + 36 + \dots + n^2\right)$
9