Panel 1

l'Hospital's Rule

Thm: Suppose f, g are diff'ble at $x=c$ and $f(c)=g(c)=0$

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

Also if $\lim_{x\to\infty} f(x)=\infty$ and $\lim_{x\to\infty} g(x)=\infty$

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

Recall: $\displaystyle\lim_{x\to\infty}\frac{5x^2-7x+1}{7x^3+9x^2-7} = \lim_{x\to\infty}\frac{10x-7}{21x^2+18x} = \lim_{x\to\infty}\frac{10}{42x+18} = 0$

Panel 2

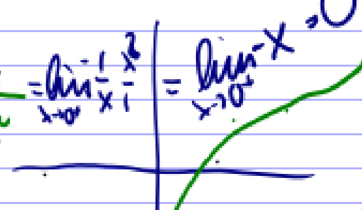
$$\lim_{x\to 1}\frac{\ln(x)}{x-1}\,\frac{0}{0} = \lim_{x\to 1}\frac{1/x}{1} = 1$$

$$\lim_{x\to\infty}\frac{e^x}{x^2}\,\frac{\infty}{\infty} \overset{\text{l'Hospital}}{=} \lim_{x\to\infty}\frac{e^x}{2x}\,\frac{\infty}{\infty} \overset{\text{l'Hospital}}{=} \lim_{x\to\infty}\frac{e^x}{2} = \frac{\infty}{2} = \infty$$

$$\lim_{x\to\infty}\frac{\ln(x)}{\sqrt[3]{x}}\,\frac{\infty}{\infty} \overset{\text{l'Hospital}}{=} \lim_{x\to\infty}\frac{1/x}{\frac{1}{3x^{2/3}}} \overset{\text{algebra}}{=} \lim_{x\to\infty}\frac{1}{x}\cdot\frac{3x^{2/3}}{1} = \lim_{x\to\infty}\frac{3}{x^{1/3}} = 0$$

$0\cdot(-\infty)$

$$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+}\frac{\ln(x)}{1/x}\,\frac{-\infty}{\infty} = \lim_{x\to 0^+}\frac{1/x}{-1/x^2} = \lim_{x\to 0^+}\frac{-1}{x}\cdot\frac{x^2}{1} = \lim_{x\to 0^+} -x = 0$$



$$= \lim_{x\to 0^+}\frac{x}{1/\ln(x)}\,\frac{0}{0} = \lim_{x\to 0^+}\frac{1}{\frac{1}{x(\ln(x))^2}} = \lim_{x\to 0^+} x(\ln x)^2$$

Panel 3

To find $\lim_{x\to c} f(x)g(x)$, if $\lim_{x\to c} f(x)=0$ and

$\lim_{x\to c} g(x)=\infty$

$\Rightarrow \lim_{x\to c} \frac{g(x)}{1/f(x)}$ $\frac{\infty}{\infty}$ l'Hospital applies

or

$\lim_{x\to c} \frac{f(x)}{1/g(x)}$ $\frac{0}{0}$ l'Hospital applies!

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Panel 4

Ex: Sketch the graph of $y = xe^x$

① Domain: all reals

② Asympt. no vert.

$\lim_{x\to \infty} xe^x = \infty$

$\lim_{x\to -\infty} xe^x = \lim_{x\to-\infty} \frac{e^x}{1/x} \frac{0}{0} = \lim_{x\to-\infty} \frac{e^x}{-1/x^2}$ got more diff.

$= \lim_{x\to-\infty} \frac{x}{1/e^x} = \lim_{x\to-\infty} \frac{x}{e^{-x}} \frac{\infty}{\infty} = \lim_{x\to-\infty} \frac{1}{-e^{-x}} = 0$

horiz. asympt. $y=0$ (on the left side only)

③ Critical: $f'(x) = e^x + xe^x = e^x(1+x)$, $x=-1$ critical

④ Infl.: $f''(x) = e^x + e^x + xe^x = e^x(2+x)$, $x=-2$ infl

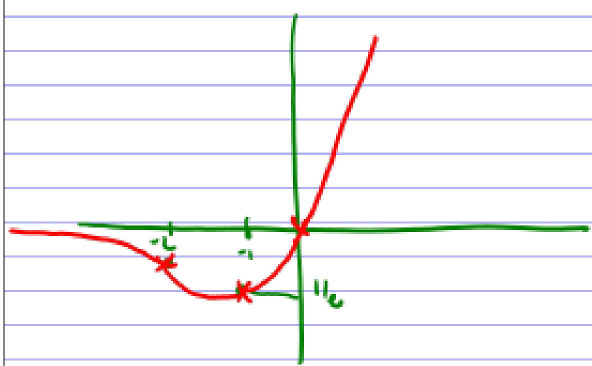
Panel 5

		-2	-1	
f'	$-$		$-$	$+$
f''	$-$		$+$	$+$
f				

$f'(x)=e^x(1+x)$

$f''(x)=e^x(2+x)$ always +

$f(x)=xe^x \quad f(-1)=\frac{1}{e},\ f(-2)=\frac{1}{e^2}$



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Panel 6

Antiderivatives

Derivative: Start with $f \Rightarrow$ find f' — Rules: Q, Prod. Chain Rule,

Antiderivative: start with f. Find F such that $F'=f$

Ex: $f(x)=3x^2 \Rightarrow F(x)=x^3$ is antiderivative because $F'(x)=3x^2=f$ so it checks out!

Antiderivatives: Guess work

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Panel 7

Find anti derivative for

$f(x) = \cos(x)$ $F(x) = \sin(x)$ $\Rightarrow F' = \cos(x)$

$f(x) = \sin(x)$ $F(x) = -\cos(x)$ $F' = \sin(x)$

difficult !!!!

$f(x) = \ln(x)$ $F(x) = x\ln(x) - x$, $F'(x) = \ln(x) + x\frac{1}{x} - 1 = \underline{\ln(x)}$

$f(x) = \frac{1}{x}$ $F(x) = \ln(x)$ $F' = 1/x$

$f(x) = x^{10}$ $F(x) = \frac{1}{11} x^{11}$ $F' = \frac{1}{\cancel{11}} \cancel{11} x^{10} = x^{10}$

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Panel 8

Ex: Find the antiderivative of

no anti-quotient rule!

$$y(x) = 4\sin(x) + \frac{2x^5 - \sqrt{x}}{x} =$$

$$= 4\sin(x) + 2\frac{x^5}{x} - \frac{\sqrt{x}}{x} = \boxed{4\sin(x) + 2x^4 - x^{-1/2}}$$

$$F(x) = -4\cos(x) + \frac{2}{5}x^5 - 2x^{1/2}$$

$$F'(x) = 4\sin(x) + \frac{2}{\cancel{5}}\cancel{5}x^4 - \cancel{2}\frac{1}{\cancel{2}}x^{-1/2}$$

<u>Rule:</u> Anti-Power Rule: $f(x) = x^p$ has anti-derivative

$$F(x) = \frac{1}{p+1} x^{p+1}, \quad p \neq -1$$

(Note: $f(x) = x^{-1} \to F(x) = \ln(x)$)

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Panel 9

Memorize

More antiderivatives

$f(x) = \sec^2(t)$ $\quad F(x) = \tan(x)$

$f(x) = \sec(x) \cdot \tan(x)$ $\quad F(x) = \sec(x)$

$f(x) = \frac{1}{\sqrt{1-x^2}}$ $\quad F(x) = \sin^{-1}(x)$

$f(x) = e^x$ $\quad F(x) = e^x$

$f(x) = 1/_{1+x^2}$ $\quad F(x) = \tan^{-1}(x)$

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Panel 10

Find a function f s.t.

$f'(x) = e^x + 20(1+x^2)^{-1}$ and $F(0) = 2$

initial condition

$= e^x + 20 \frac{1}{1+x^2}$

$F(x) = e^x + 20 \tan^{-1}(x) + C$

$\Rightarrow F'(x) = e^x + 20 \frac{1}{1+x^2}$

$F(0) = e^0 + 20 \tan^{-1}(0) + C \overset{\text{want}}{=} 2$

$1 + C = 2 \Rightarrow \underline{C = 1}$

Answer: $F(x) = e^x + 20 \tan^{-1}(x) + 1$

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Panel 11

Ex: If $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$ and $f'(1) = 1$

Find $f(x)$.

$f''(x) = 12x^2 + 6x - 4$

$f'(x) = 4x^3 + 3x^2 - 4x + C$

Want $f'(1) = 1 = 4 + 3 - 4 + C = 3 + C \Rightarrow -2$

$f'(x) = 4x^3 + 3x^2 - 4x - 2$

$f(x) = x^4 + x^3 - 2x^2 - 2x + D$

$f(0) = 4 = D \Rightarrow$

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