Panel 1

Review

2.5 Chain Rule
2.6 Implicit Diff
2.7. Related Rates
2.9 Differentials and Errors
4.1 Max / Min
Curve Sketching
Optimization
MVT + Rolle

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Panel 2

$$f(x) = \frac{x^2 \cos(1-x)}{(1-2x)^2}$$

$$f'(x) = \frac{(\quad \mathrm{I} \quad)(1-2x)^2 - x^2\cos(1-x)((\mathrm{II}) \quad)}{(1-2x)^4}$$

I: $2x\cos(1-x) + x^2\left(-\sin(1-x)\cdot(-1)\right)$

II: $2(1-2x)\cdot(-2)$

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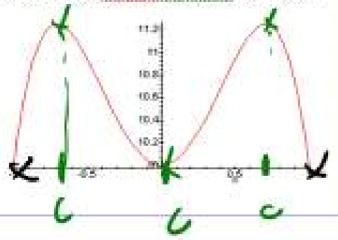
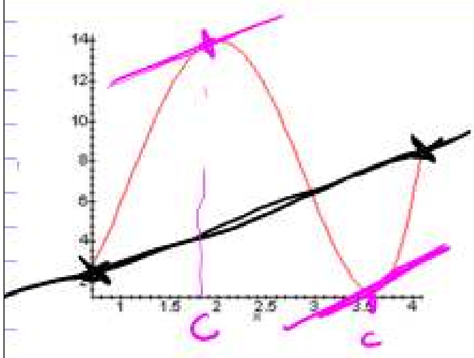
Panel 3

State Rolle's theorem. Then look at the graph numbers c that Rolle's theorem mentions.



$f'(c)=0$



MVT:

$\frac{f(b)-f(a)}{b-a} = f'(c)$

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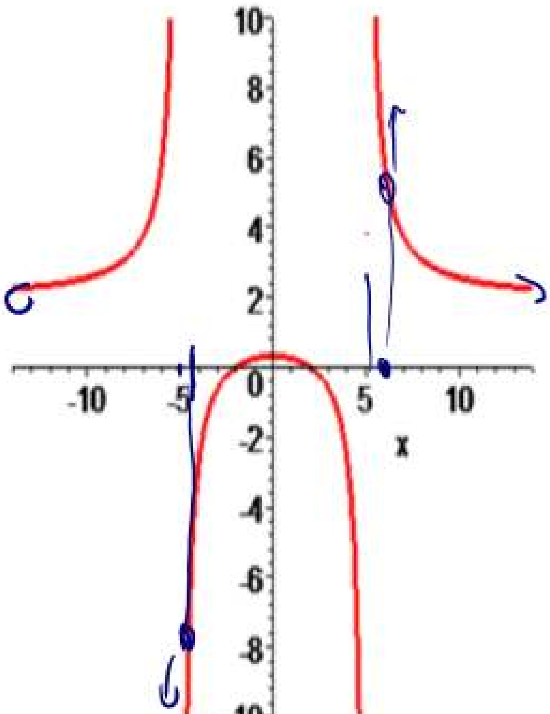
Panel 4



For the functic

a) $\lim_{x\to\infty} f(x) = 2$

b) $\lim_{x\to-\infty} f(x) = 2$

c) $\lim_{x\to 5^+} f(x) = \infty$

d) $\lim_{x\to -5^+} f(x) = -\infty$

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Panel 5

$y^3 - 5x^2 = 3x$

$\frac{d}{dx}(y^3 - 5x^2) = \frac{d}{dx}(3x)$

$3y^2 y' - 10x = 3$

Find the slope of the tangent line to the graph of $y^4 + 3y - 4x^3 = 5x + 1$ at the point (1, -2).

$4y^3 y' + 3y' - 12x^2 = 5$

$4(-8)y' + 3y' - 12 = 5$

$-32y' + 3y' = 17$

$y' = \frac{17}{29}$

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Panel 6

$f(x) = x^4 - 2x^2$ Incr./decr.

① f'

② critical

③ table of f', f

$f'(x) = 4x^3 - 4x = 0$

$4x(x^2-1) = 0 \Rightarrow x = \pm 1, 0$

	-1	0	1	
f'	$-$	$+$	$-$	$+$
f	↘	↗	↘	↗

incr

decr

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Panel 7

$f(x) = x^5 - 5x^3$. Concavity

① f' $\qquad f'(x) = 5x^4 - 15x^2 = 5x^2(x^2-3) = 0$

② $f' = 0$ $\qquad x = 0, \pm\sqrt{3}$

③ table f'', f

	$-\sqrt{3}$	0	$\sqrt{3}$	
f''	+	−	−	+
f	$\cup$	$\cap$	$\cap$	$\cup$

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Panel 8

$f(x) = \frac{x}{x^2+1}$ on $[0,3]$ Abs. Extrema.

$f' = \frac{1(x^2+1) - x\,2x}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2} = 0 \qquad x = \pm 1$

never zero

critical $x = +1$, $\cancel{-1}$ not in $[0,3]$

	x	$f(x)$	
	1	1/2	← max
end points	0	0	← min
end points	3	3/10	

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Panel 9

Gas is escaping from a spherical balloon at a rate of 10 ft^3/hr. At what rate is the radius changing when the volume is 400 ft^3.

$$V(r) = \frac{4}{3}\pi r^3 \quad \Rightarrow 400 = \frac{4}{3}\pi r^3 \Rightarrow r^3 = 400 \cdot \frac{3}{4\pi}$$

$$\cancel{V'(r) = \frac{4}{3}\pi\, 3r^2} \qquad \cancel{V'(r) = \frac{dV}{dt}}$$

$$r^3 = \frac{300}{\pi}$$

$$r = \sqrt[3]{\frac{300}{\pi}}$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$V'(t) = \frac{4}{3}\pi\, 3r^2 \cdot r' \quad \Rightarrow V' = 4\pi r^2 r'$$

$$-10 = 4\pi(\quad)^2 r'$$

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Panel 10

$$f(x) = \frac{1}{(1+2x)^4} \approx f'(c)(x-c) + f(c) \qquad c = 0 \text{ Given} \qquad f(0) = 1$$

$$f(x) = (1+2x)^{-4}$$

$$f'(x) = -4(1+2x)^{-5} \cdot 2$$

$$\Rightarrow f'(0) = -4/1^5 \cdot 2 = -8$$

$$\frac{1}{(1+2x)^4} \approx -8(x-0) + 1 = -8x+1$$

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Panel 11

Cube: $V = x^3$

$A = 6x^2$

$\Rightarrow dV = 3x^2\,dx = 3 \cdot 900 \cdot 0.1 = 270$

$\Rightarrow dA = 12x\,dx = 12 \cdot 30 \cdot 0.1 = 36$

errors

$dx = 0.1$, $x = 30$

rel errors: $\frac{dV}{V} = \frac{270}{30^3} = \frac{27\cancel{0}}{27000} = 0.01$

$\frac{dA}{A} = \frac{36}{6 \cdot 900} = \frac{6}{900} = \frac{2}{300} = 0.00666$

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